Revision (Mid-Term)

* Virtual Work:

$$\Delta = 1/EI \int M_0 M_1 \, dx$$

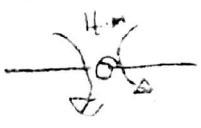
حيث:

M_0: الموسمة الـ B.M.D الناشئة نتيجة الأحمال الخارجية بطريقة الـ (Super Position)

M_1: نلغي الأحمال ونضع حمل مقداره 1t عند si مقداره 1t.m حسب المطلوب.

أنواع المطالب

- Rotation. (دوران)
  - لو طلب الـ Rotation عند نقطة نلغي الأحمال ونضع عزم مقداره 1t.m
  - لو طلب الـ relative عند نقطة تشوه ـــ وهنا هذه الحالة نضع عزمين عكس بعض مقدار كل منهم 1t.m كالآتي



- Displacement (إزاحة)
  - وفي هذه الحالة نلغي الأحمال ونضع حمل مقداره 1t رأسي ـــ نوجد الإزاحة الرأسية وأخيراً لو طلب الإزاحة الأفقية

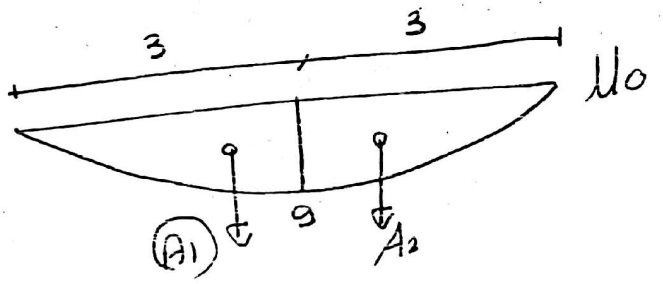
$\Delta_{rel} \leq \Delta \leq \Delta_{HL} \leq \Delta_{VL}$: Δ

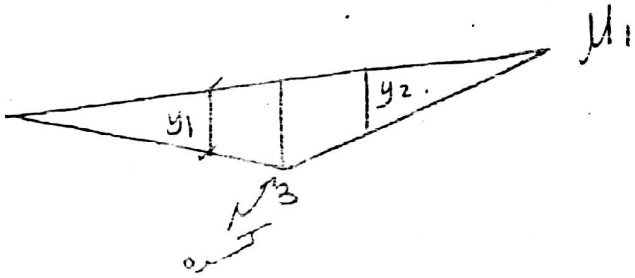
$$\therefore \Delta = 1/EI \left[\begin{array}{l} \text{يؤخذ كل تركيز حاصلة ضرب } M_0 \\ * \text{ الطول المقابل من } M_1 \\ \text{بالسهام} \end{array} \right]$$

* ملحوظات :

في حالة وجود كسر في M_1 نقوم بكتابة الأسرة في M_0 وتركيز بمقطع وسالب.

Ex:





كسرة

Consistent deformation

* الحلول :

① رسم الـ B.M.D & S.F.D للعزوم المفروضة استاتيكياً .

② حساب الـ deflection أو الـ rotation عند أحد النقاط .

* خطوات حل الحلول الأول :

1 حساب المجاهيل (R) والمعادلات (C) حيث أن المجاهيل هي عبارة عن reactions وهي عزوم أو قوى x والمعادلات هي ($\Sigma M_x, \Sigma M_y, \Sigma M$) وهي معادلات الـ Mainsystem

2 يتم اختيار الـ M.S حيث عدد المجاهيل الزائدة (Redundent) هي عبارة عن أن نحيد الـ M.S

Stable — M.S — Det

- اختيار الـ M.S :

تمثل المجاهيل عند حذف بعد اختيار ما يظهر من رد فعل داخلي

تمثل المجاهيل عند حذف ما يعيق الحركة Reaction

R=3, C=2, Red=1

M.S

3) نضع $X_1 = 0.0$ وندرسم B.M.D نتيجة الأحمال الخارجية بطريقة الـ SuperPosition (M_0) .

4) نلغي الأحمال ونضع $X_1 = H$ Si $X_1 = 1t.m$ وندرسم الـ (M_1) .

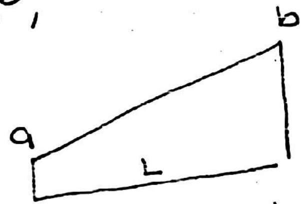
5) بالتعويض لإيجاد الـ (X_1)

$$\delta_{10} + X_1 \delta_{11} = 0.0$$

$$\delta_{10} = 1/EI \int M_0 M_1 dx$$

$$\delta_{11} = 1/EI \int M_1 M_1 dx$$

ولإيجاد الـ δ_{11} بدلاً من تكامل المساحة * البعد المقابل الذي نفسه عند تحديد يتم تطبيق القوانين الآتية .



$$\delta_{11} = l/3 (a^2 + b^2 + ab)$$

$$\delta_{11} = l/3 (a)^2$$

$$\delta_{11} = l (a)^2$$

6) نضع X_1 ككل على الـ M وندرسم B.M.D & S.F.D مرة أخرى

- من الممكن أن يطلب الـ B.M.D فقط وفي هذه الحالة الأسهل بدلاً من إيجاد الـ reaction لكان نحسب العزوم عند أي نقطة كالآتي :

$$M_f = M_0 + X_1 M_1$$

* هذه الطريقة أن تقوم بإلغاء عيد حدده من الدرجة الثانية وفي هذه الحالة يتم حذف محولين عند اختيار الـ (S.S) فيصبح لدينا (X_1 ، X_2)

في هذه الحالة نقوم بخطوات كالآتي:

1) نضع $X_1 = X_2 = 0$ ونرسم B.M.D بطريقة (Superposition) (الأحمال)

2) نأخذ الأحمال ونضع $X_1 = 1t$ ، $X_2 = 0$ ونرسم M_1

3) نأخذ " ونضع $X_2 = 1t$ ، $X_1 = 0$ ونرسم M_2

4) التعويض في المعادلات الآتية لإيجاد X_1 & X_2

$$\delta_{10} + X_1 \delta_{11} + X_2 \delta_{12} = 0$$
$$\delta_{20} + X_1 \delta_{21} + X_2 \delta_{22} = 0$$

Where,

$$\delta_{10} = 1/EI \int M_0 M_1 dx$$
$$\delta_{20} = 1/EI \int M_0 M_2 dx$$
$$\delta_{11} = 1/EI \int M_1 M_1 dx$$
$$\delta_{22} = 1/EI \int M_2 M_2 dx$$
$$\delta_{12} = \delta_{21} = 1/EI \int M_1 M_2 dx.$$

← بالقوانين بجموعها

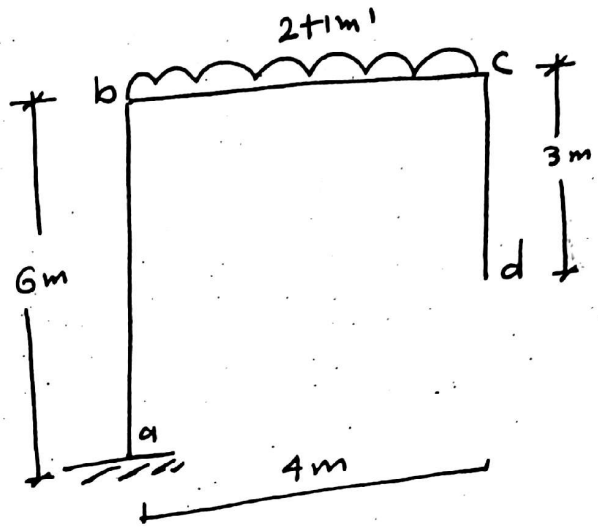
ثم نجد أو or.

$$M_f = M_0 + X_1 M_1 + X_2 M_2$$

(5

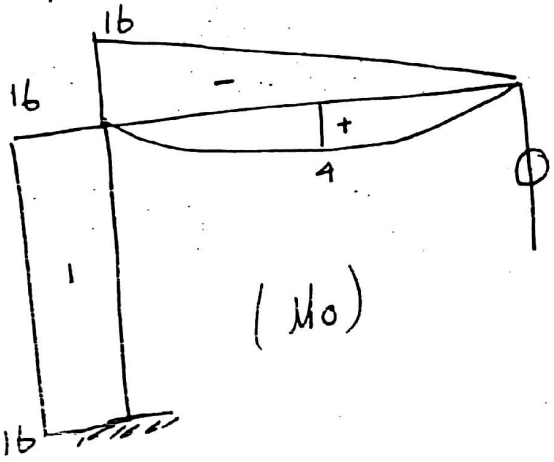
Ex:1 (virtual work):

Compute the horizontal and vertical deflection at Point (d) of the frame shown if $EI = 20000\ t.m^2$.



Sol

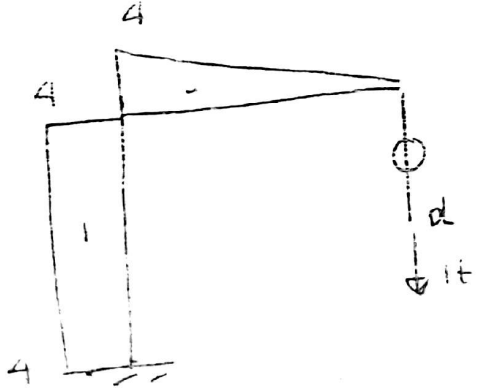
[1] نرسم الـ B.M.D تتيجة الأحمال الخارجية (Mo).



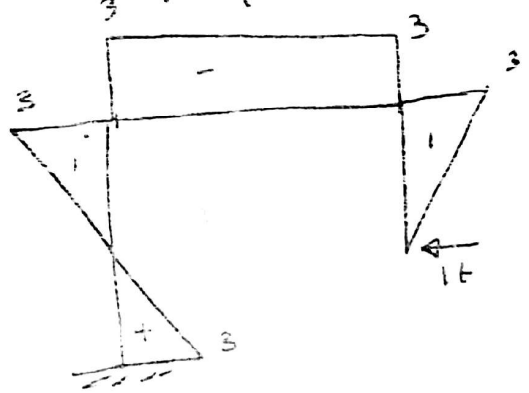
(Mo)

[2] نأثر الأحمال على الجملة بالحلول لنوع عن مقدار 1t

$M_1 (\delta_{VLd})$



$M_1 (\delta_{HLd})$



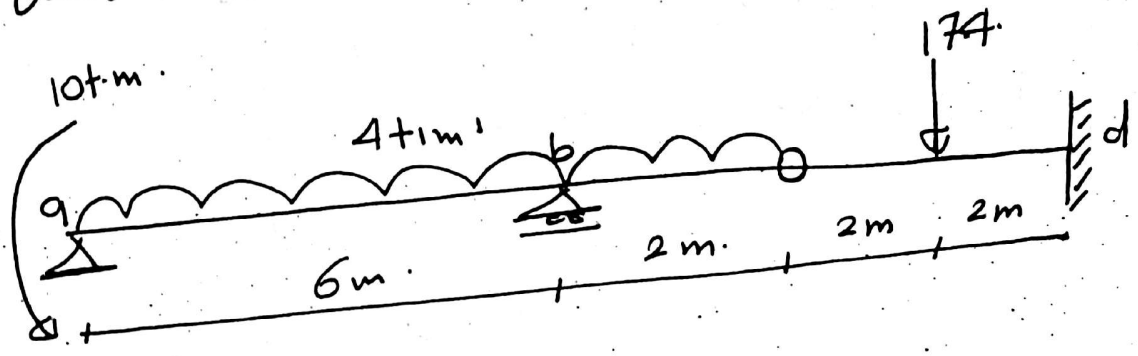
$$\Delta_{1Ld} = 1/EI \left[\begin{array}{l} (0.5 * 4 * 16 * \frac{2}{3} * 4) - \\ (2/3 * 4 * 4 * 2) \\ + (16 * 6 * 4) \end{array} \right]$$

$$= \frac{490.67}{EI}$$

$$\Delta_{1Ld} = 1/EI \left[\begin{array}{l} (0.5 * 4 * 16 * 3) \\ -(2/3 * 4 * 4 * 3) \end{array} \right]$$

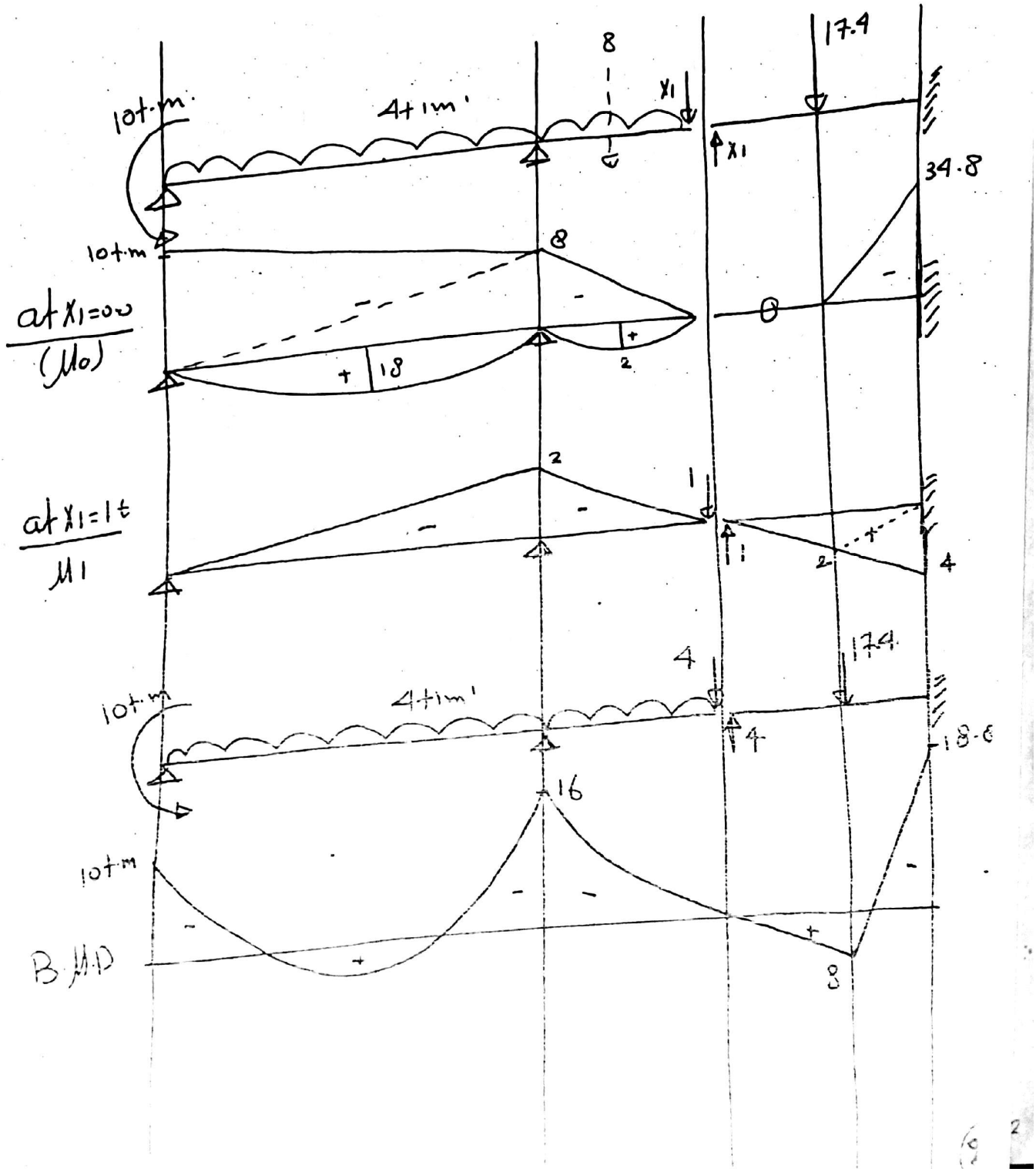
$$= \frac{64}{EI}$$

EX:3 Using Consistent deformation, Draw The B.M.D diagrams For The Shown beam.



$r = 4$
$C = 2+1 = 3$
$1 \text{ red} = 1$

Sol



$$\delta_{10} = 1/EI \left[+(0.5*6*8*(4/3)) + (0.5*6*10*2/3) - (2/3*6*18*1) + (0.5*2*8*4/3) - (2/3*2*2*1) - (1/2*2*34.8*(3.33)) \right]$$

$$= \frac{-128}{EI}$$

$$\delta_{11} = 1/EI \left[6/3(2)^2 + 2/3(2)^2 + 4/3(16) \right] = \frac{32}{EI}$$

$$\delta_{10} + X_1 \delta_{11} = 0.0$$

$$\frac{-128}{EI} + X_1 \left(\frac{32}{EI}\right) = 0.0$$

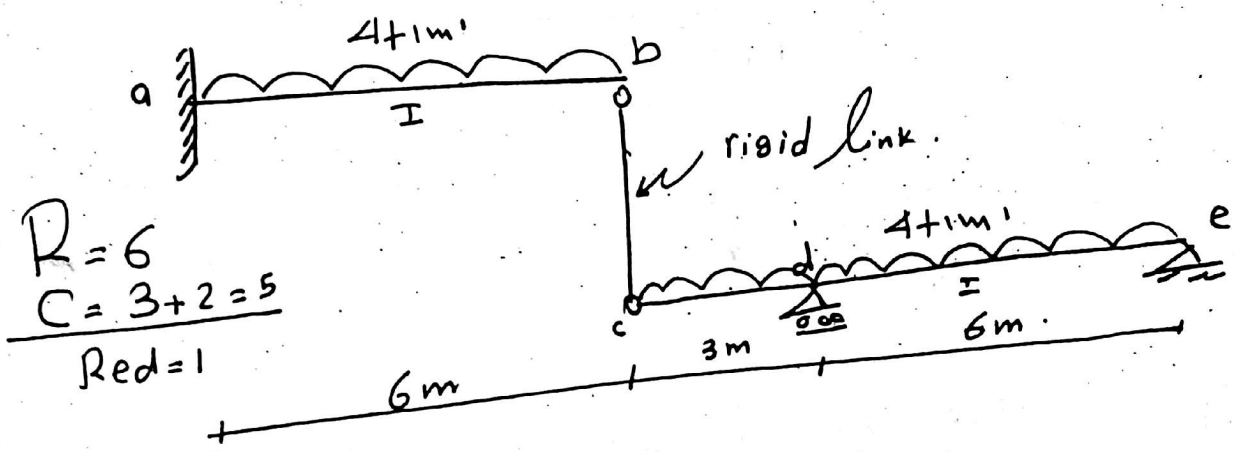
$$\therefore X_1 = 4\ t$$

$$\therefore M_f = M_0 + X_1 M_1$$

$$\therefore M_f = M_0 + 4M_1$$

ونضرب X_1 في M_1 على الـ S.D وبالتالي نعمل أوفلي بالطريقة العادية ده شكل للعطل بالكامل

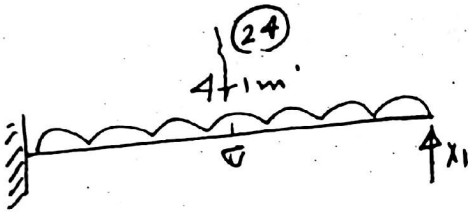
Ex: Draw B.M.D & S.F.D for The Shown beam using Consistent deformation:

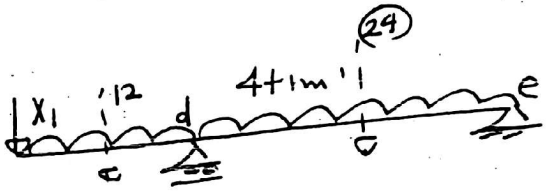


R = 6
C = 3+2 = 5
Red = 1

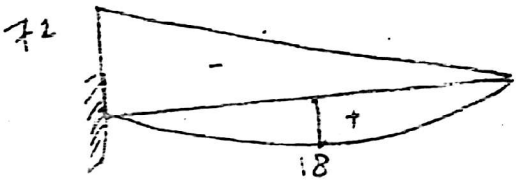
scl

M.S:

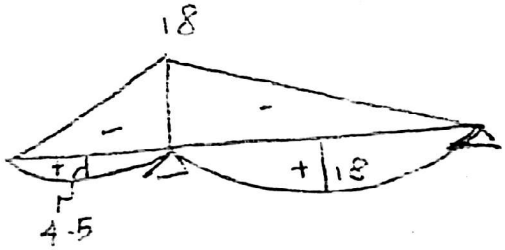




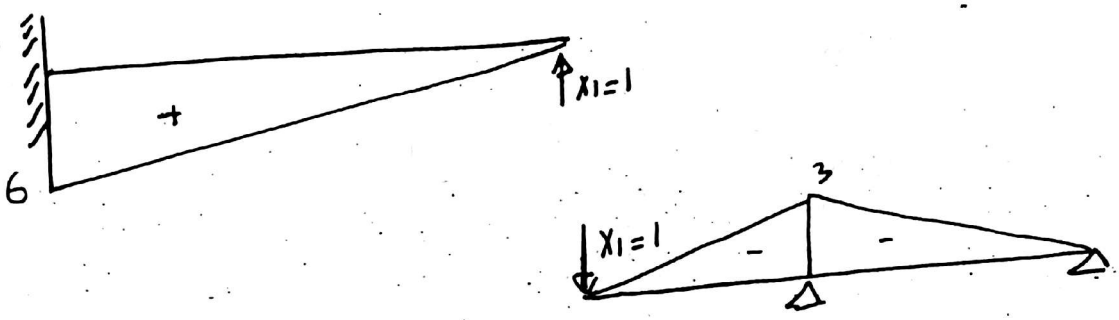
at $X_1 = 0.0$, M_0



M_0



at $X_1 = H$, M_1 :



$$\delta_{10} = 1/EI \left[\begin{array}{l} -(0.5*6*72*4) + (2/3*6*18*3) \\ +(0.5*3*18*2) - (2/3*3*4.5*1.5) \\ +(0.5*6*18*2) - (2/3*6*18*1.5) \end{array} \right]$$

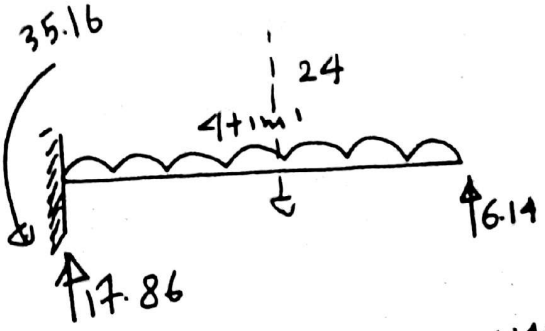
$$= \frac{-607.5}{EI}$$

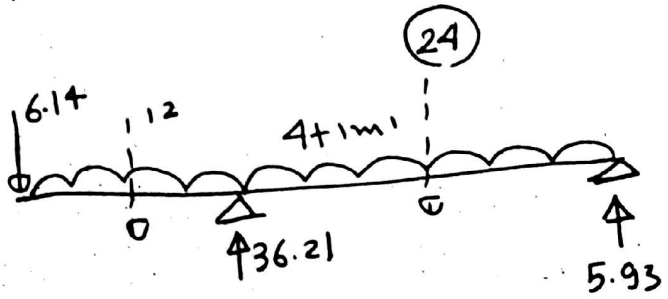
$$\delta_{11} = 1/EI \left[\begin{array}{l} 6/3(36) + 3/3(9) \\ \quad + 6/3(9) \end{array} \right] = \frac{99}{EI}$$

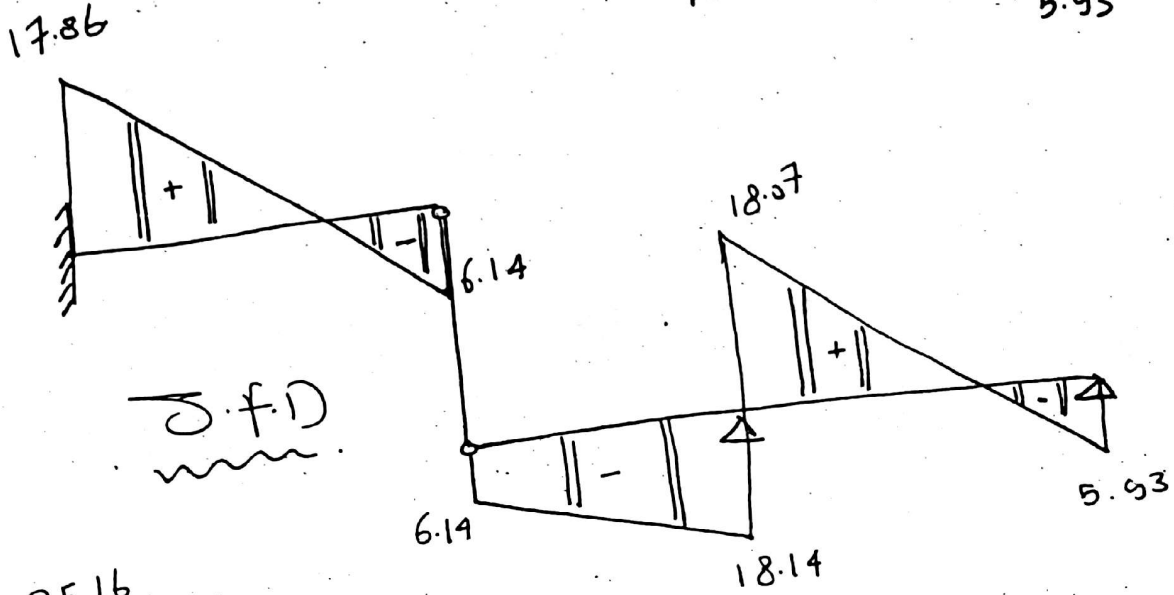
$$\delta_{10} + X_1 \delta_{11} = 0.0$$

$$\frac{-607.5}{EI} + X_1 \frac{99}{EI} = 0.0$$

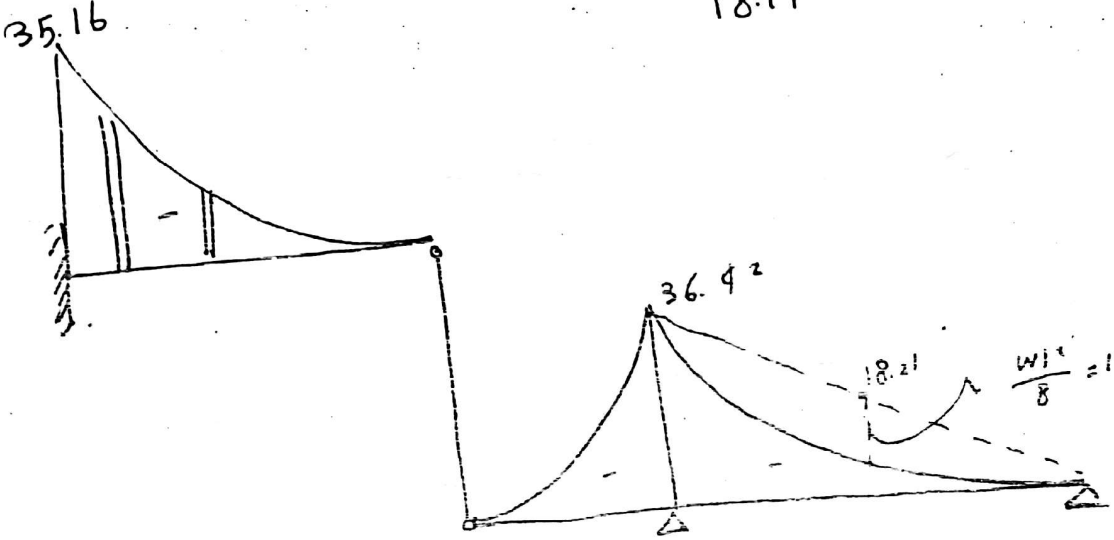
$$\therefore X_1 = 6.14 \text{ t}$$







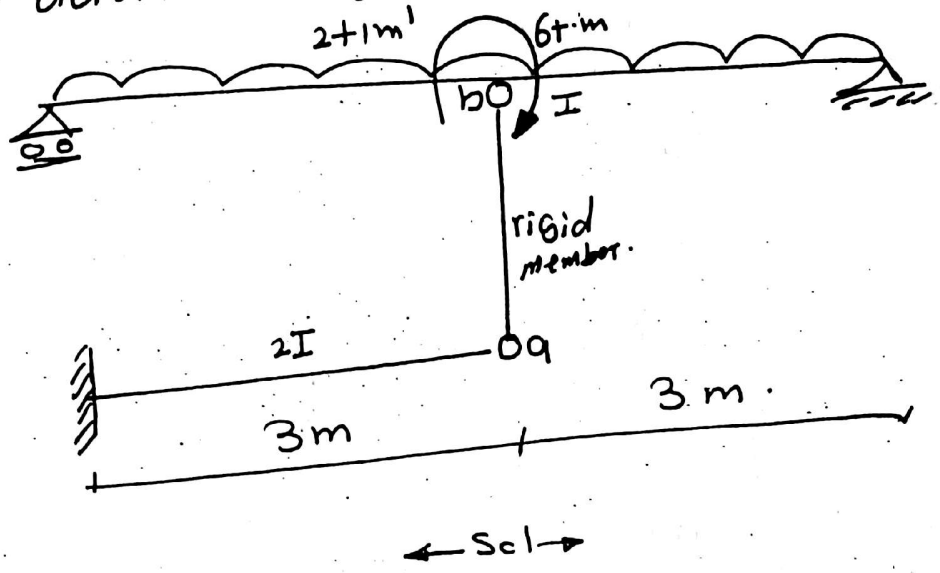
S.f.D



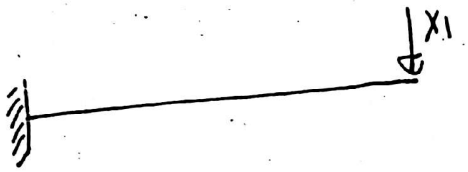
B.M.D

Ex:

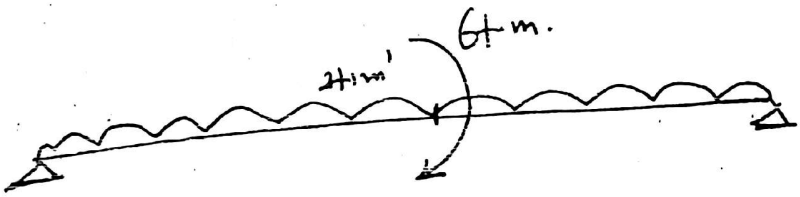
for the given structure, draw the B.M.D
- determine the deflection at Point (a).



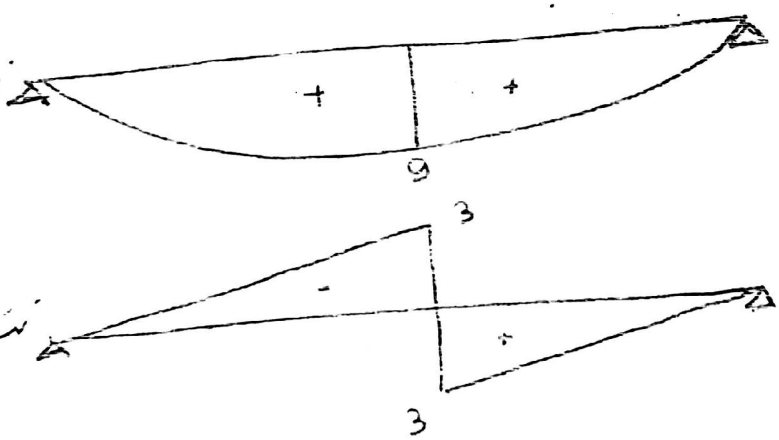
M.S:

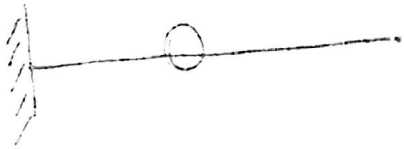


at $X_1 = 0.0$, M_0

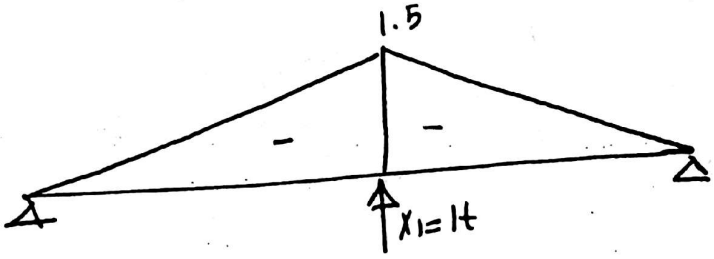


$M_0 =$ + (تدريجة بلوزة) + (تدريجة الفتح)

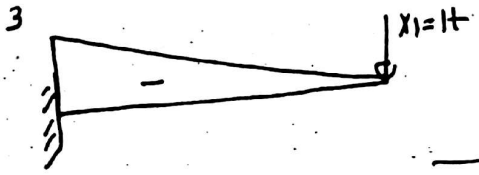




at $X_1 = 1t$, M_1:



(M_1)



$$\delta_{10} = 1/EI \left[(2/3 * 3 * 9 * 2) * (5/8 * 1.5) \right]$$

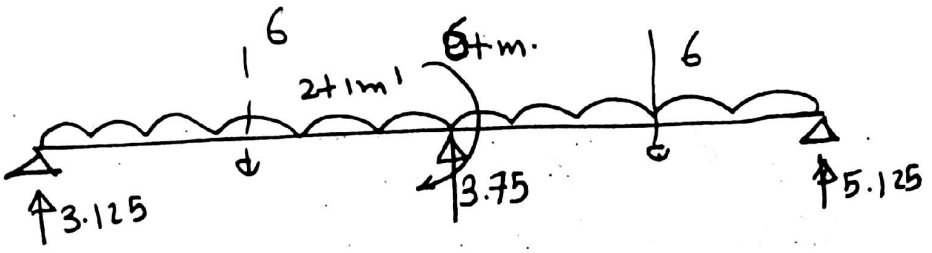
$$= \frac{-33.75}{EI}$$

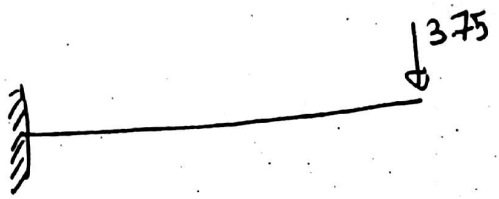
$$\delta_{11} = 1/EI \left[3/3(1.5)^2 * 2 + \frac{3/3(9)}{2} \right] = 9/EI$$

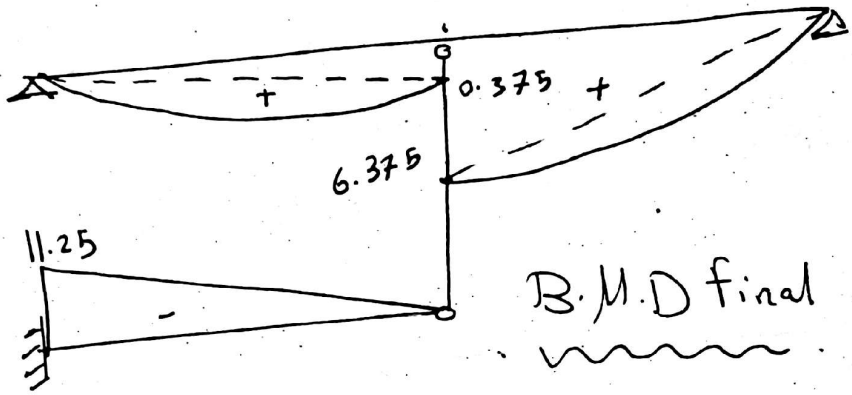
$$\delta_{10} + X_1 \delta_{11} = 0$$

$$\frac{-33.75}{EI} + X_1 \frac{9}{EI} = 0$$

$$\therefore X_1 = +3.75t$$







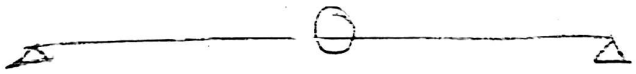
B.M.D final

⇒ find deflection at a ::

نلغي الاحمال ونضع ١ $X_1=0.0$ ونضع عمل عند a باتجاه $1t$

لأسفل عند النقطة a ونرسم M_1^*

$$\therefore \delta_a = 1/EI \int M_f M_1^* dx.$$



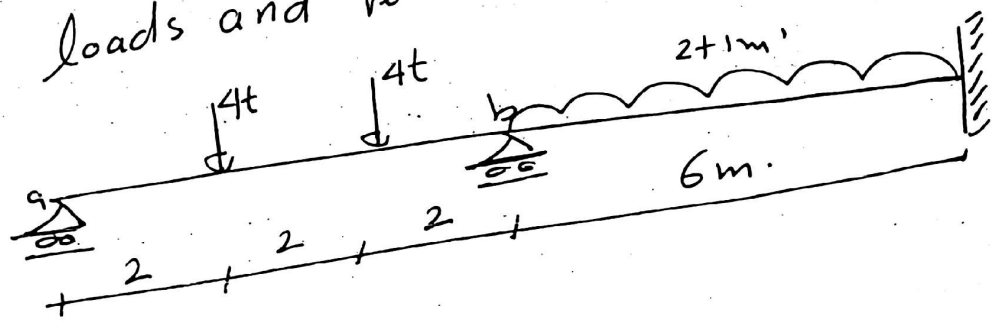
$$\delta_a = \frac{1}{2EI}\left[0.5 * 3 * 11.25 * 2\right] = \frac{16.875}{EI}$$

EX: Using 3.Mom:

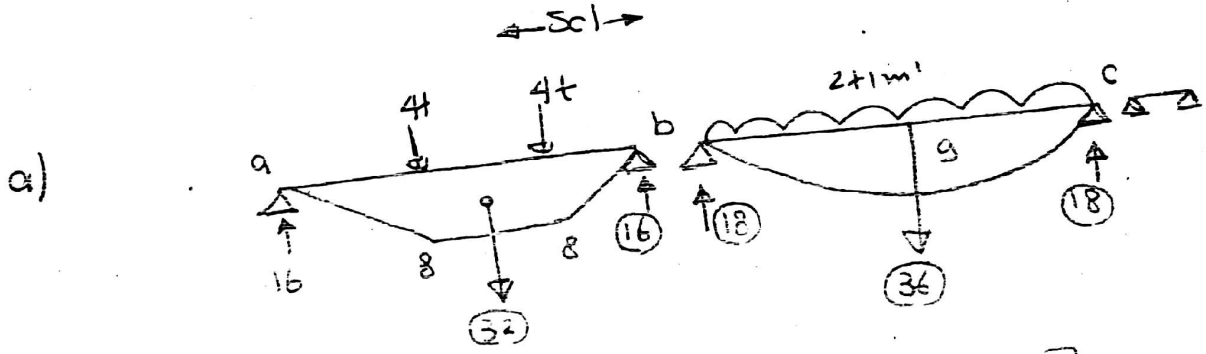
a) draw B.M.D due to the applied loads.

b) in the absence of loads Draw B.M.D due to vertical settlement at b = 1 cm ($EI = 6000\ m^2.t$).

c) Find the required vertical settlement at support (b) such that the negative moment at support b equal the maximum Positive moment Through the Span ab due to the applied loads and vertical settlement at support (b).



a)



3.Mequ at b: $0.0 + 2M_b(6+6) + M_c(6) = -6[16+18]$

$24M_b + 6M_c = -204$ → (1)

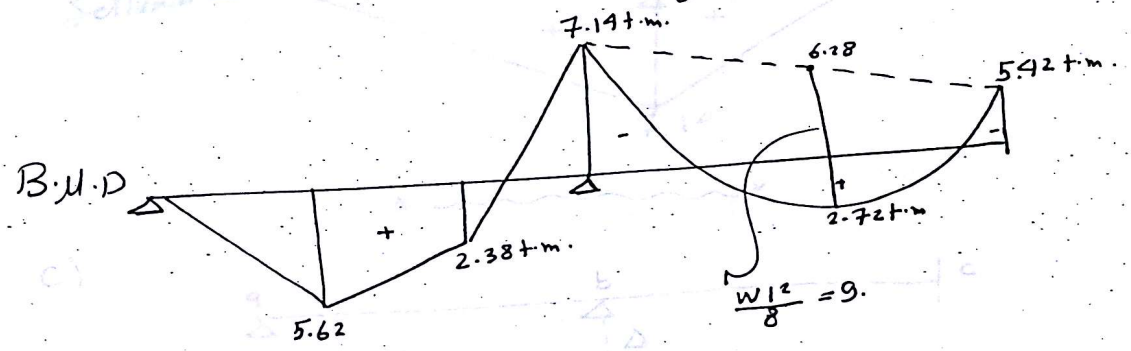
3.Mequ at c: $M_B^{(6)} + 2M_c(6+0) + 0.0 = -6[18+0]$

$6M_b + 12M_c = -108$ → (2)

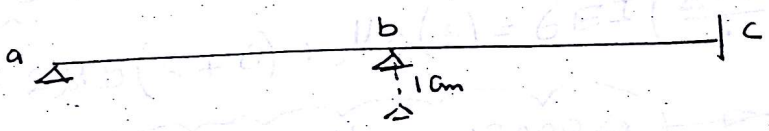
(16)

by solving ① & ② get $M_b = -7.14$ t·m.

$M_c = -5.42$ t·m.



b)



3 Mequ at b:-

$$0.0 + 2M_b\left(\frac{6}{I} + \frac{6}{I}\right) + M_c(6/I) = 6E\left(\frac{0.01-0}{6} + \frac{0.01-0.0}{6}\right)$$

$$24M_b + 6M_c = 6EI\left(\frac{0.02}{6}\right)$$

$$= 6 * 6000 * \frac{0.02}{6} = 120$$

$$\therefore 24M_b + 6M_c = 120 \rightarrow (1)$$

3 Mequ at c:-

$$M_b(6) + 2M_c(6+0) + 0 = 6EI\left(\frac{0-0.01}{6}\right)$$

$$6M_b + 12M_c = -60 \rightarrow (2)$$

$M_b = +7.14$, $M_c = -8.57$

3. Memat b:

$$Ma(6) + 2Ma(6+6) + 0.0 = -6(4P_1 + 32)$$

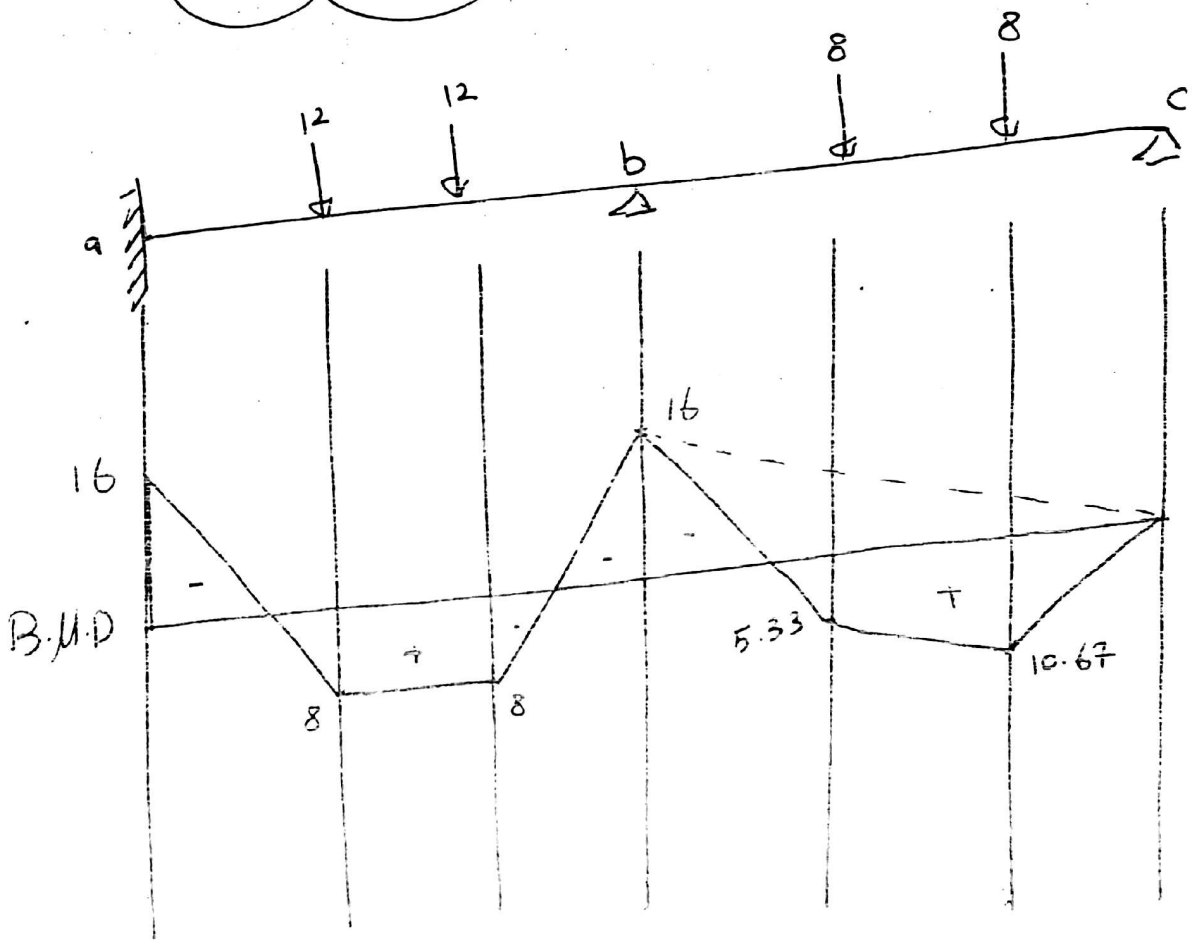
$$6Ma + 24Ma = -24P_1 - 192$$

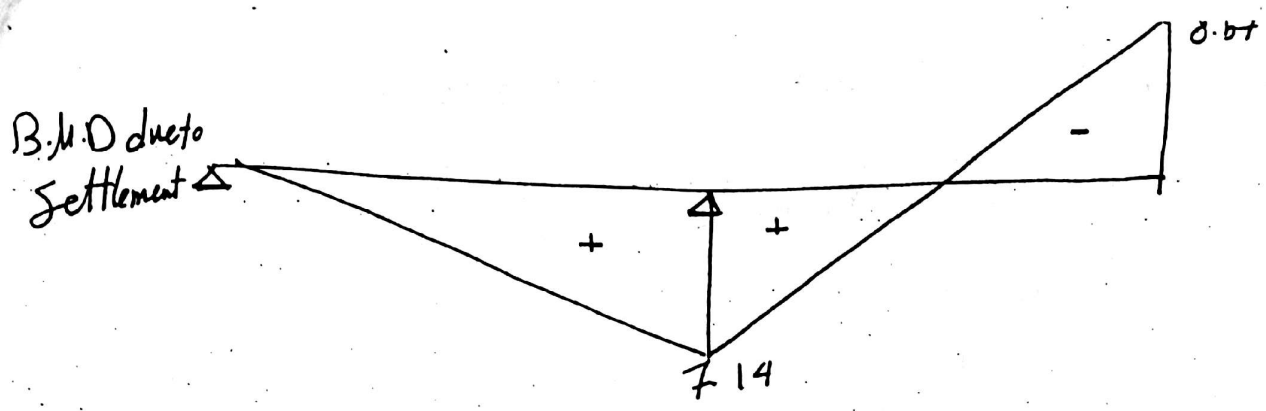
$$-40P_1 + 24P_1 = -192$$

$$\therefore +16P_1 = +192$$

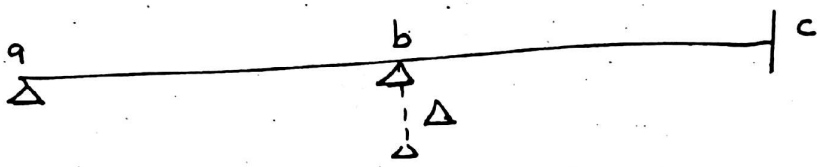
$$\therefore P_1 = 12t$$

$$\therefore Ma = Mb = -16t\cdot m$$





c)



3. Mequat b:

$$0.0 + 2M_b(6+6) + M_c(6) = 6EI\left(\frac{\Delta - 0}{6} + \frac{\Delta - 0}{6}\right)$$

$$24M_b + 6M_c = 12000\Delta \rightarrow (1)$$

3. Mequat c:

$$M_b(6) + 2M_c(6+0) + 0.0 = 6EI\left(\frac{0 - \Delta}{6} + 0\right)$$

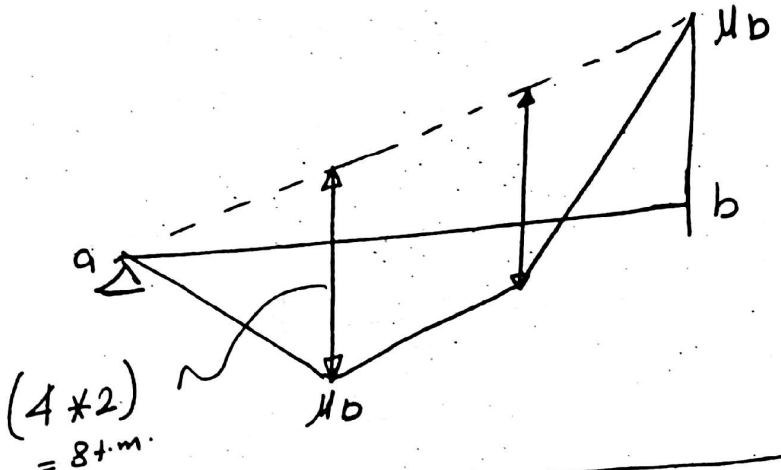
$$6M_b + 12M_c = -6000\Delta \rightarrow (2)$$

by solving (1) & (2) Sol

$$M_b = 714.28\Delta$$

$$M_c = -857.14\Delta$$

∵ Negative moment at B = Max +ve B.M in span ab due to (loads and settlement at b).



∴ $M_b = 8 - 1/3 M_b$ ∴ $M_b = 6$ t.m

→ ∵ B.M.D due to loads only = −7.14.

∴ B.M.D due to settlement at b = +1.14.

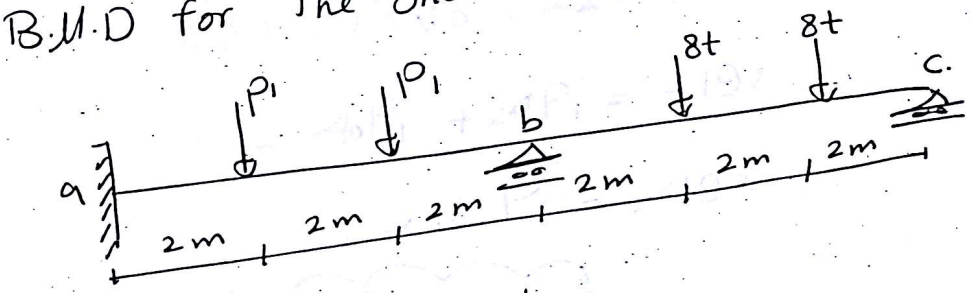
حيث أن مجموع العزم السالب = −6 t.m حقارة العزم بسالب النقطة عند b نتيجة الأحمال والهبوط معاً.

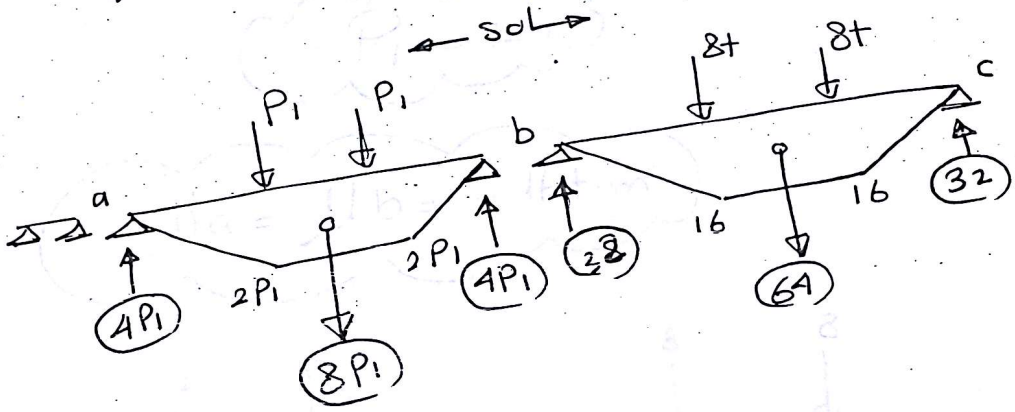
∵ M_b due to settlement = 714.28 Δ

∴ 714.28 Δ = 1.14

∴ $\Delta = 1.596 * 10^{-3}$ m

Ex:2 Using 3.Moment eqn. Find The Value of P_1 such That The B.M at (a) equals The B.M at b in Magnitude and sign ($M_a = M_b$), Further draw B.M.D for The shown beam.





3.Mat a::

$$0.0 + 2M_a(0.0+6) + M_b(6) = -6(0+4P_1)$$

$$12M_a + 6M_b = -24P_1$$

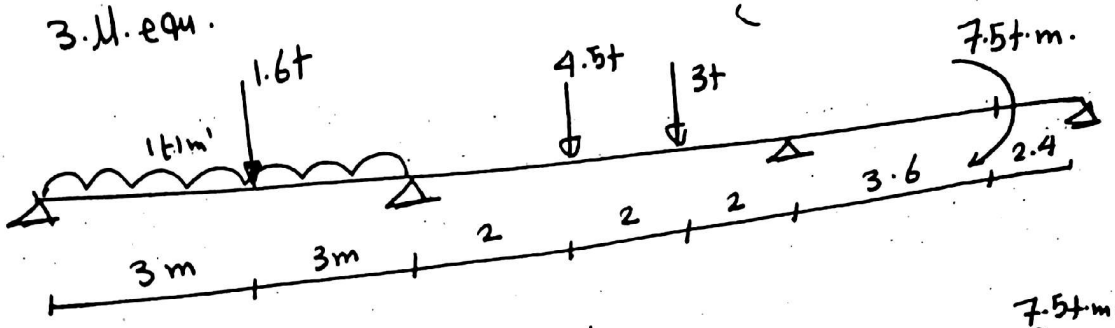
but $M_a = M_b$

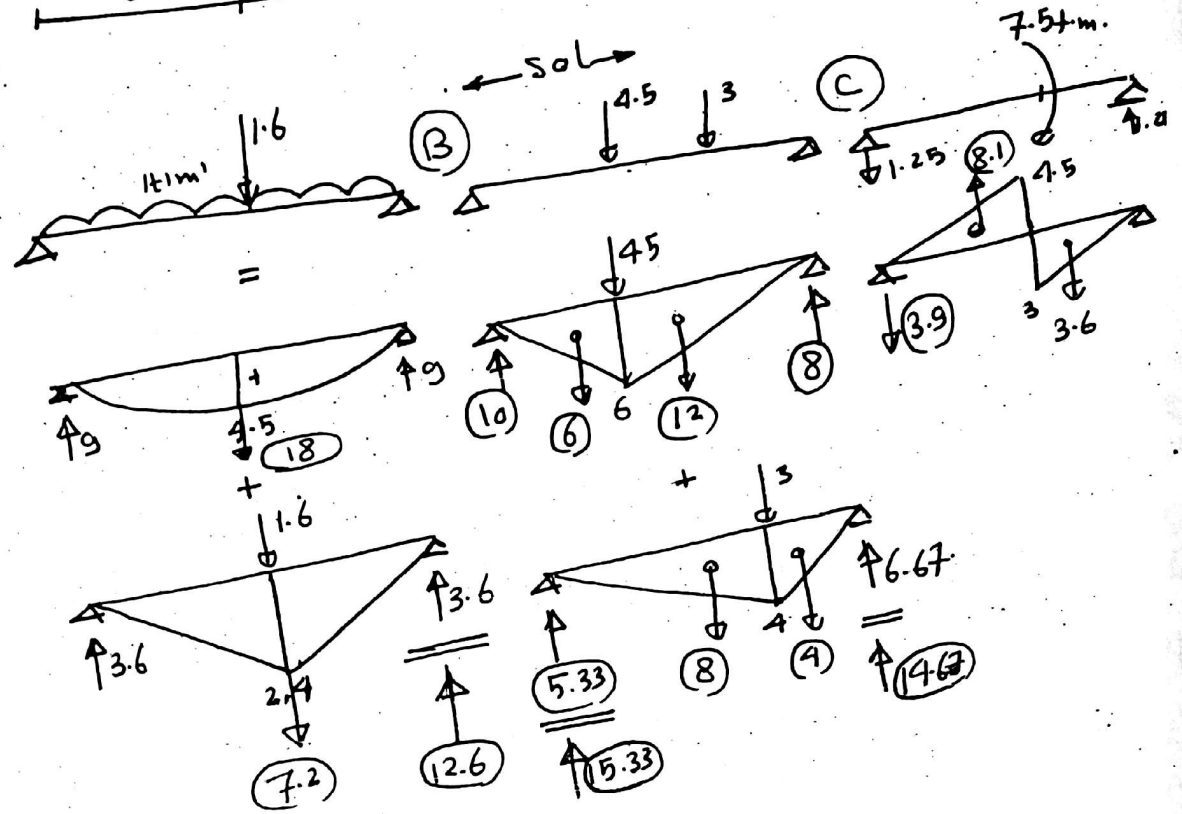
$$\therefore 18M_a = -24P_1$$

$$\therefore M_a = -1.33P_1$$

(20

Ex:1 Draw S.f.D & B.M.D for the shown beam using 3.M.eqn.





3. Meqn at B:

$$0.0 + 2M_B(6+6) + M_C(6) = -6(12.6 + 15.33)$$

$$24M_b + 6M_C = -167.58 \quad \rightarrow (1)$$

3. Meqn at C:

$$M_B(6) + 2M_C(6+6) + 0.0 = -6\left[14.67 + (-3.9)\right]$$

$$6M_b + 24M_C = -64.62 \quad \leftarrow (2)$$

by solving (1) & (2) get

$M_B = -6.73$ t·m

$M_C = -1.01$ t·m