CONCRETE SOCIETY

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Post-tensioned Concrete Floors – Design Handbook

Report of a Concrete Society Working Party

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CONCRETE SOCIETY
TECHNICAL REPORT

POST-TENSIONED CONCRETE FLOORS – DESIGN HANDBOOK

This Technical Report was prepared by a Working Party of the Society's Design Group which is one of the specialist technical groups within The Technical Development Centre.

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NOTATION

A_c	Area of concrete
A_{ps}	Area of prestressing tendons
A_s	Area of un-tensioned reinforcement
A_{su}	Area of shear reinforcement
a	Drape of tendon
b	Width of section
b_v	Breadth of member; or for T-, I- and L-beams the breadth of the rib
d	Effective depth of tension reinforcement or tendons
d_n	Depth to the centroid of the compression zone
E_c	Short-term modulus of elasticity of concrete
E_{ci}	Modulus of elasticity of concrete at time of transfer
E_{ps}	Modulus of elasticity of prestressing tendons
E_s	Modulus of elasticity of un-tensioned reinforcement
e	Eccentricity
f_{cc}	Compressive stress in concrete at extreme fibre used to calculate serviceability un-tensioned reinforcement requirements
f_{ci}	Concrete strength at (initial) transfer (in N/mm^2)
f_{co}	Stress in concrete at the level of the tendon due to initial prestress and dead load (in N/mm^2)
f_{ct}	Tensile stress in concrete at extreme fibre used to calculate serviceability un-tensioned reinforcement requirements
f_{cu}	Characteristic concrete cube strength (in N/mm^2)
f_{pb}	Tensile stress in tendons at (beam) failure (in N/mm^2)
f_{pe}	Effective prestress (in tendon) after all losses (in N/mm^2)
f_{pu}	Characteristic strength of prestressing steel (in N/mm^2)
f_t	Maximum design principal tensile stress ($=0.24\sqrt{f_{cu}}$, in N/mm^2)
f_y	Characteristic strength of bonded un-tensioned reinforcement (in N/mm^2)
h	Overall depth of section
I	Second moment of area
L	Effective span length
l	Length of tendon
l'	Length of tendon affected by wedge draw-in
M_A	Moment due to applied loads
M_o	Moment necessary to produce zero stress in the concrete at the extreme tension fibre
M_s	Secondary moment due to prestress
M_u	Ultimate resistance moment
P	Prestress force
p'	Slope of prestress force profile
P_k	Characteristic strength of tendon (in kN)
P_o	Prestressing force in the tendon at the jacking end
P_x	Prestressing force at distance x from jack
s	Distance between points of contra-flexure in tendon
u	Length of a critical shear perimeter
V	Shear force due to ultimate loads
V_c	Ultimate shear resistance of concrete
V_{co}	Ultimate shear resistance of a section uncracked in flexure
V_{cr}	Ultimate shear resistance of a section cracked in flexure
V_{eff}	Design effective shear force
v	Design shear stress at cross-section
v_c	Design concrete shear strength
w	Uniformly distributed load
x	Neutral axis depth
y_o	Half the side of the prestress end block
y_{po}	Half the side of the prestress anchor loaded area
Z_t	Top section modulus

z_b	Bottom section modulus
α	Angle change in tendon from anchor to point considered (radians)
α'	Average angle change in tendon per unit length (radians/metre)
Δ	Wedge draw-in
μ	Coefficient of friction
ϕ	Creep coefficient
γ_f	Partial safety factor for load
γ_m	Partial safety factor for material strength
ϖ	Prestressed tendon 'wobble' factor (radians/metre)

1. INTRODUCTION

The use of post-tensioned concrete floors in buildings has been consistently growing in recent years. The greatest use of this type of construction has been in the USA, and in California it is the primary choice for concrete floors. Post-tensioned floors have also been used in Australia, Hong Kong, Singapore and Europe. Their use in the UK is now increasing rapidly.

Typical applications have been:

Offices
Car parks
Shopping centres
Hospitals
Apartments
Industrial buildings

These are illustrated in Figures 1, 2 and 3.

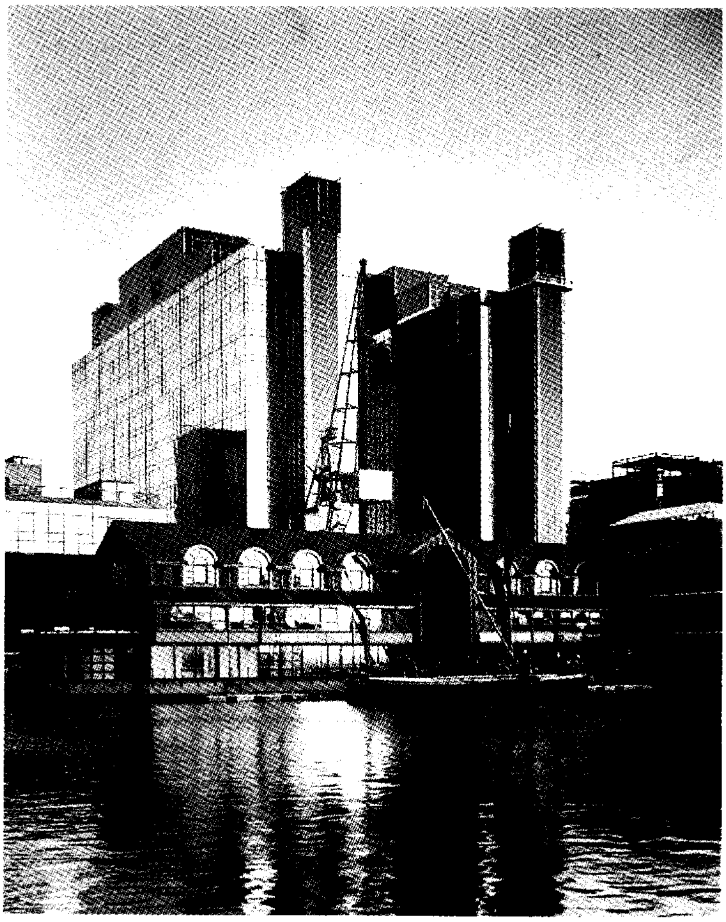


Figure 1: Harbour Exchange Tower.

Figure 2: Grosvenor Square Car Park - Southampton.

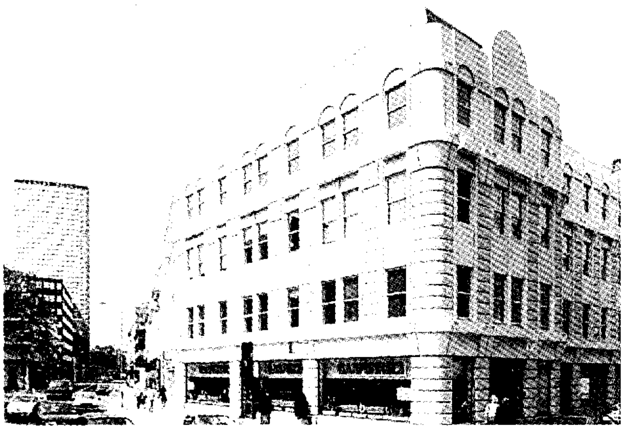


Figure 3: New Oxford Street.

The Concrete Society has published three reports on this subject, Technical Report No 8[1], The Design of Post-Tensioned Concrete Flat Slabs in Buildings; Technical Report No 17[2], Flat Slabs in Post-Tensioned Concrete with Particular Regard to the Use of Unbonded Tendons - Design Recommendations; Technical Report No 25[3], Post-Tensioned Flat-Slab Design Handbook. TR17 was a revision of TR8, and TR25 amplified the recommendations of TR17. The purpose of the current report is to update the information contained in TR17 and TR25 in line with BS8110, 1985[4], to combine these two reports into one document and to expand some of the recommendations in line with current practice.

This report explains the overall concept of post-tensioned concrete floor construction as well as giving detailed design recommendations. The intention is to simplify the tasks of the designer and contractor enabling them to produce effective and economic structures. Post-tensioned floors are not complex. The techniques, structural behaviour and design are simple and very similar to reinforced concrete structures. The prestress tendons provide a suspension system within the slab and the simple arguments of the triangle of forces apply with the vertical component of the tendon force carrying part of the dead and live loading and the horizontal component reducing tensile stresses in the concrete. Two design examples are given in Appendix A.

The report is intended to be read in conjunction with BS8110[4]. Those areas not covered in BS8110[4] are described in detail in the report with references given as appropriate. The principles laid out in the report may also be applied

to designs in accordance with Eurocode EC2[5], but some of the details will need to be modified.

Two other Concrete Society publications give useful background information to designers of post-tensioned floors. They are: Technical Report No. 21[6], Durability of Tendons in Prestressed Concrete and Technical Report No. 23[7], Partial Prestressing.

It should be noted that since the integrity of the structure depends on a relatively small number of prestressing tendons and anchorages the effect of workmanship and quality of materials can be critical. This should be understood by all parties involved in both design and construction.

1.1 Advantages of post-tensioned floors

The main advantages of post-tensioned floors over conventional reinforced concrete in-situ floors, may be summarised as follows:

Increased clear spans
Thinner slabs
Lighter structures
Reduced cracking and deflections
Reduced storey height
Rapid construction
Better watertightness

These advantages can result in significant savings in overall costs. There are also some situations where the height of the building is limited, in which the reduced storey height has allowed additional storeys to be constructed within the building envelope.

1.2 Structural types considered

The report is primarily concerned with suspended floors. However, the recommendations apply equally well to foundation slabs except that since the loads are generally upward rather than downward the tendon profiles and locations of un-tensioned reinforcement are mirrored.

The types of floor which can be used range from flat plates to one-way beam and slab structures. An important distinction between structural types is whether they span one-way or two-ways. This is discussed in greater detail in Section 2.2.

1.3 Amount of prestress

The amount of prestress provided is not usually sufficient to prevent tensile stresses occurring in the slab under design load conditions. The structure should therefore be considered to be partially prestressed.

The amount of prestress selected affects the un-tensioned reinforcement requirements. The greater the level of prestress, the less reinforcement is likely to be required. Unlike reinforced concrete structures, a range of acceptable designs are possible for a given geometry and loading. The optimum solution depends on the relative costs of prestressing and un-tensioned reinforcement and on the ratio of live load to dead load.

Average prestress levels usually vary from 0.7 to 2.5N/mm^2 for solid slabs and occasionally up to 6N/mm^2 for ribbed or waffle slabs. However, when the prestress exceeds approximately 2N/mm^2 or the floor is very long, the effects of restraint to slab shortening by supports may become important. If the supports are stiff a significant proportion of the prestress force goes into the supports so that the effective prestressing of the slab is reduced (see Section 3.3).

1.4 Bonded or unbonded tendons

Post-tensioned floors can be constructed using either bonded or unbonded tendons. The relative merits of the two techniques are subject to debate. The following points may be made in favour of each:

Bonded:
- develops higher ultimate flexural strength
- does not depend upon the anchorage after grouting
- localises the effects of damage

Unbonded:
- provides greater available lever arm
- reduces friction losses
- simplifies prefabrication of tendons
- grouting not required
- can be constructed faster
- generally cheaper

1.5 Analytical techniques

The design process is described in Section 6. The analytical techniques are the same as those used for reinforced concrete structures. The structure is normally subdivided into a series of equivalent frames upon which the analysis is based. These frames can be analysed using moment distribution or other hand techniques, however it is now more common to make use of a plane frame computer analysis program. In addition to standard plane frame programs, there are available a number of programs, specifically written for the design of prestressed structures. These programs reduce the design time but are not essential for the design of post-tensioned floors. For more complicated flat slabs or for those which are be repeated many times, a grillage or finite element analysis of the floor may be more appropriate.

2. STRUCTURAL BEHAVIOUR

2.1 Effects of prestress

The primary effects of prestress are a pre-compression of the floor and an upward load within the span which balances part of the downward dead and live loads. In a reinforced concrete floor, tensile cracking of the concrete is a necessary accompaniment to the generation of economic stress levels in the reinforcement. In post-tensioned floors both the pre-compression and the upward load in the span act to reduce the tensile stresses in the concrete. However, the level of prestress is not usually enough to prevent all tensile cracking under full design live loading at Serviceability Limit State. Under reduced live load much of the cracking will not be visible.

The act of prestressing causes the floor to bend, shorten, deflect and rotate. If any of these effects are restrained, secondary effects of prestress are set up. As stated above, if the level of prestress does not exceed approximately $2 N/mm^2$ the secondary effects due to the restraint to shortening are usually neglected. However, unless the floor can be considered to be statically determinate, the displacements of the floor sets up secondary moments which cannot be neglected.

Secondary effects are discussed in more detail in Section 6.9 and the calculation of these effects is described in Appendix D.

2.2 One-way and two-way spanning floors

There are several different types of post-tensioned floor. Some of the more common layouts are given in Figures 4, 5, 6, 7 and 8. An important distinction between types of floors is whether they are one-way or two-way spanning structures.

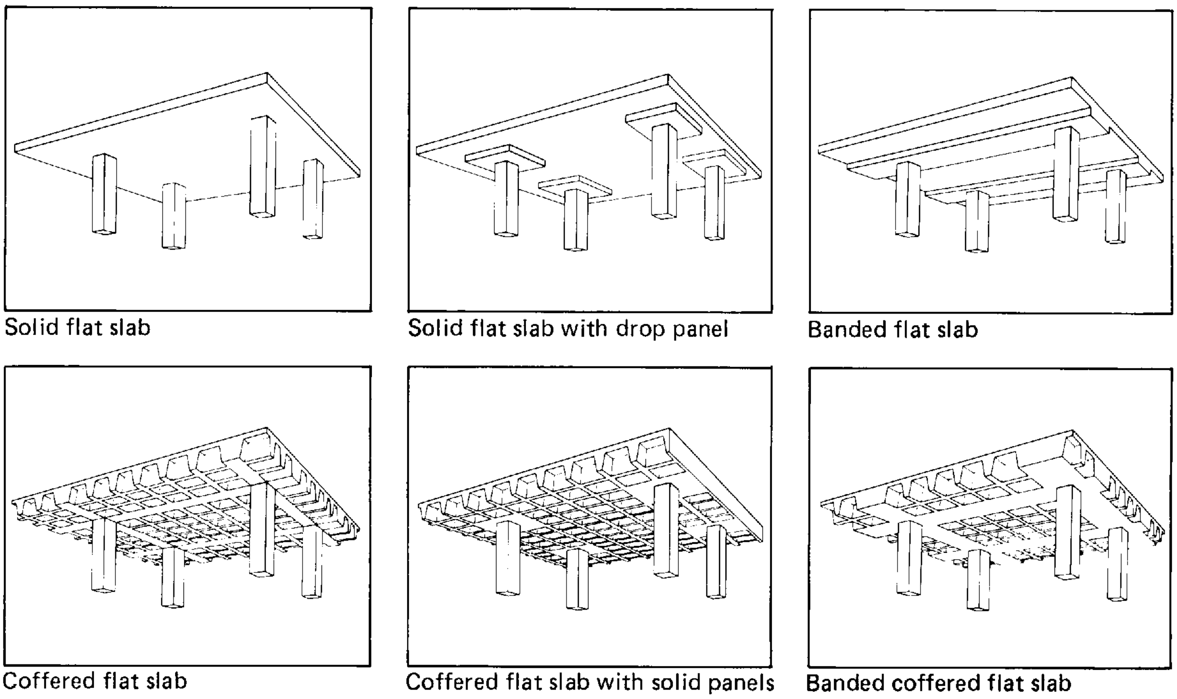
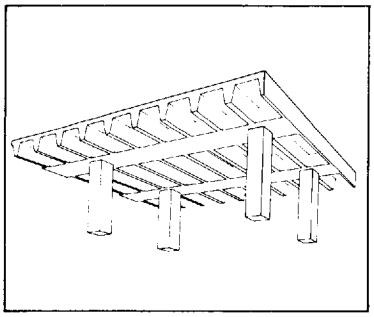
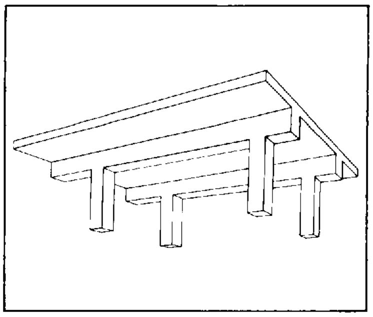


Figure 4: Typical flat slabs.
Note: See Section 2.4 for limiting criteria of two-way action.



Ribbed slab



Beam and slab

Figure 5: Typical one-way spanning floors.

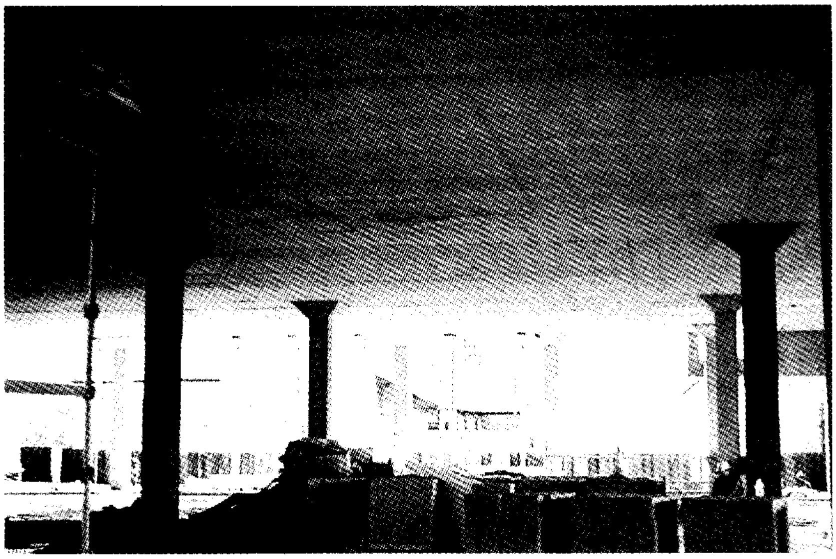


Figure 6: Post-tensioned flat slab.

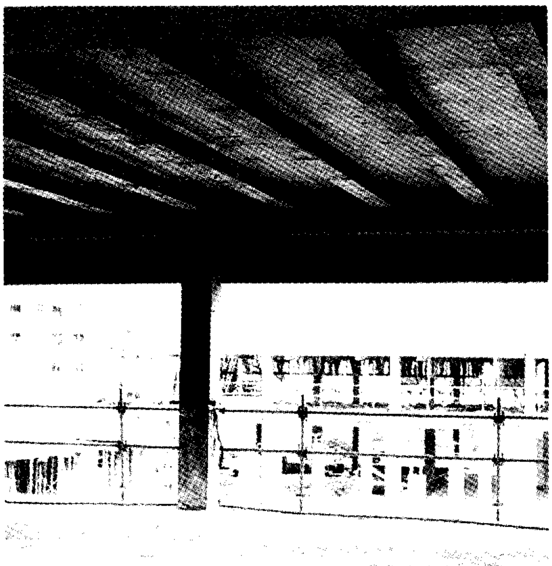


Figure 7: Post-tensioned ribbed slab.

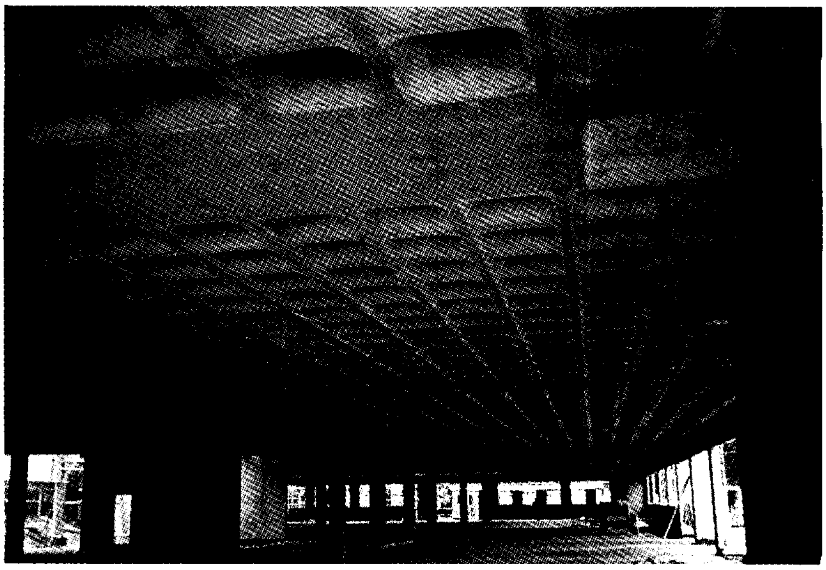


Figure 8: Post-tensioned coffered slab.

One-way floors carry the applied loading primarily in one direction and are treated as beams or plane frames. On the other hand, two-way spanning floors have the ability to sustain the applied loading in two directions. However, for a structure to be considered to be two-way spanning it must meet several criteria. These criteria are discussed in Section 2.4.

2.3 *Flexure in one-way floors*

One-way spanning floors are usually designed as Class 3 structures in accordance with BS8110[4]. Although cracking is allowed, it is assumed that the concrete section is uncracked and that hypothetical tensile stresses can be carried at Serviceability Limit State. The allowable stresses are discussed in Section 6.10.1.

The behaviour of one-way floors at loads less than that which would cause cracking can be assumed to be linear and elastic. BS8110[4] recommends that when the tensile stresses under design permanent loads are less than the allowable stresses for Class 2 structures, then the deflection may be predicted using gross section properties. In other cases calculation of deflections should be based on the moment-curvature relationship for cracked sections.

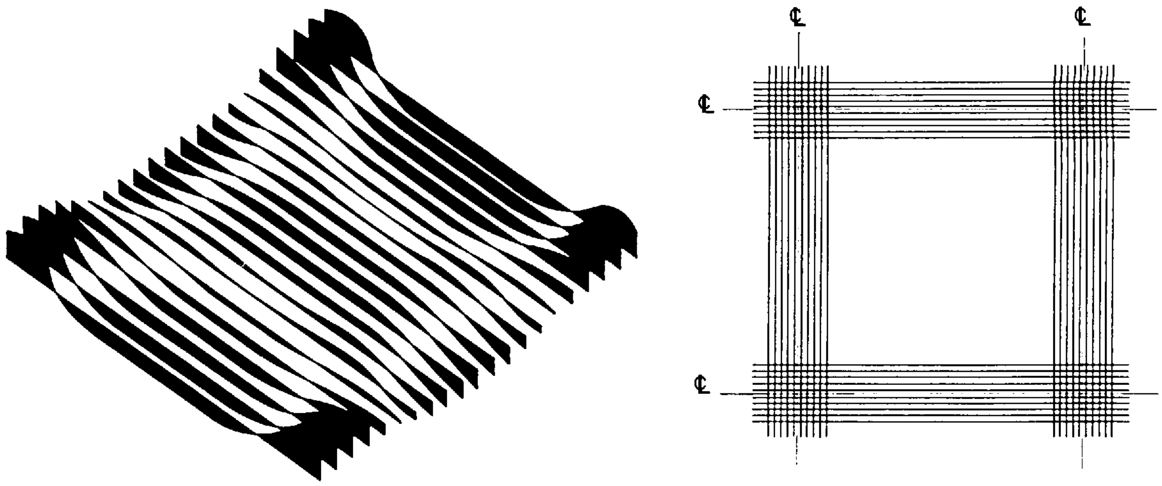
2.4 *Flexure in flat slabs* (two-way spanning)

In the context of this report, flat slabs are those floors which can carry loads in two different directions to discrete column supports. These are defined as flat slabs in BS8110[4]. It must be emphasised that these structures are not the same as two-way slabs in accordance with Section 3.5 of BS8110[4]. Two-way slabs in BS8110[4] always span on to beams or walls, i.e. continuous supports, and are not considered in this report.

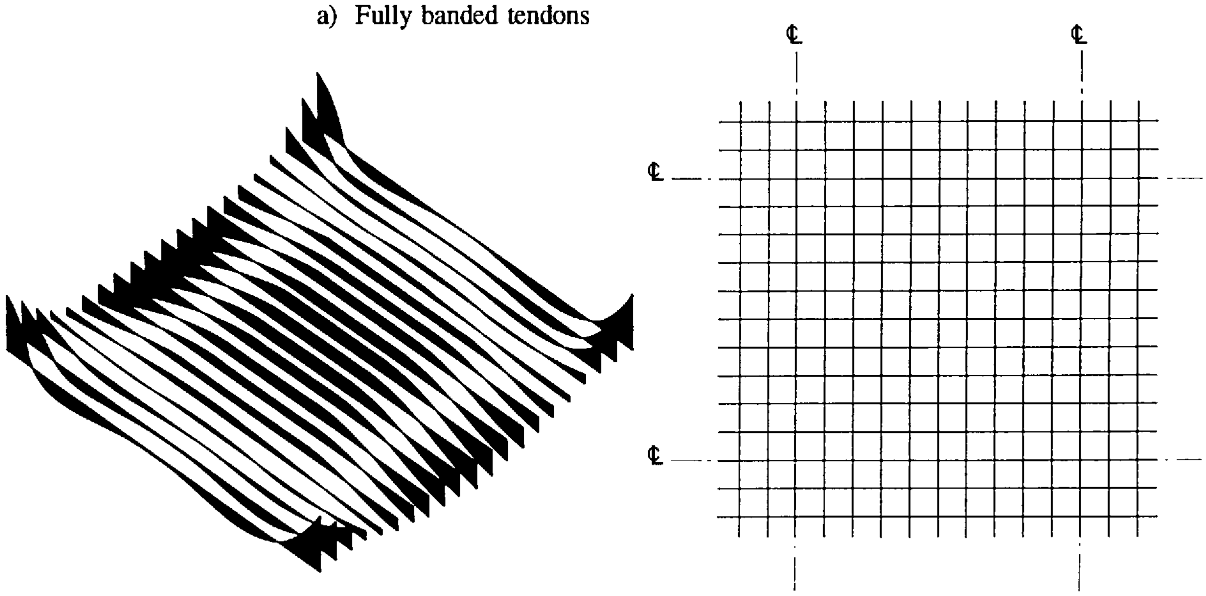
One misconception held by some engineers is to consider a reduced load when analysing the slab in one direction using the equivalent frame method. A flat slab supported on columns, rather than perimeter beams, can fail as a one-way mechanism just as a single-way slab, and it should be reinforced to resist the moment from the full load in each orthogonal direction.

Tests and applications have demonstrated that a post-tensioned flat slab behaves as a flat plate almost regardless of tendon arrangement (see Figure 9). The effects of the tendons are, of course, critical to the behaviour as they exert loads on the slab as well as provide reinforcement. The tendons exert equivalent vertical loads on the slab known as equivalent loads (see Section 6.7), and these loads may be considered like any other dead or live load. Since the tendon effect is opposite to the effect of gravity loads, the net load causing bending is reduced. An additional effect of the tendons is the axial precompression which counteracts flexural tensile stresses. Therefore, at service dead load, the net downward load causing bending in the slab is normally very low and the floor is essentially under uniform axial compression.

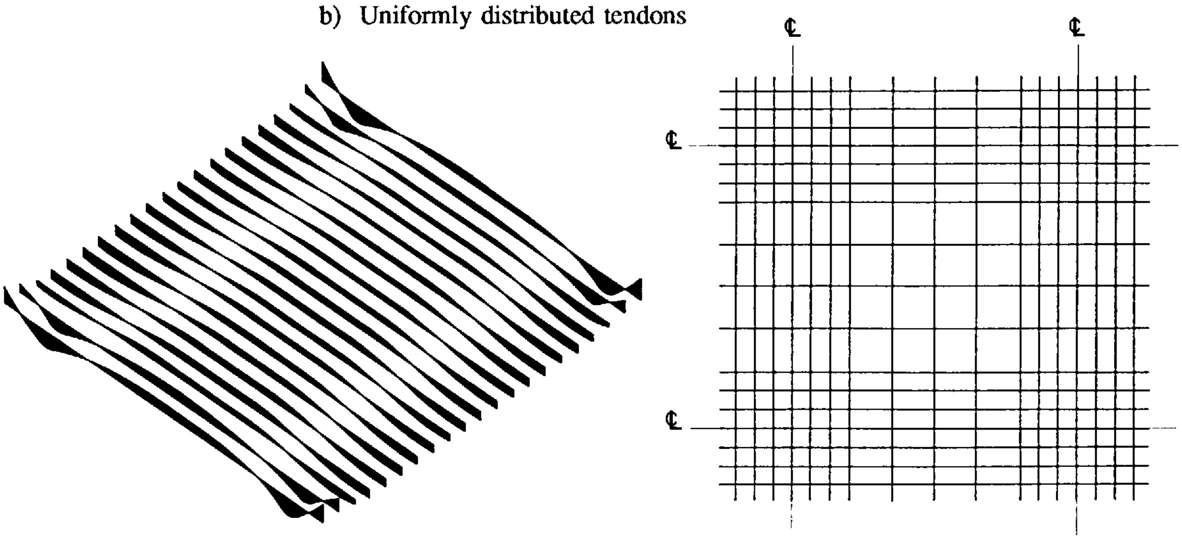
Examination of the distribution of moments for a flat plate in Figures 10 and 11 reveals that hogging moments across a panel are sharply peaked in the immediate vicinity of the column and that the moment at the column face is several times the moment midway between columns. It should be noted that the permissible stresses given in Table 2 of Section 6.10.1 are average stresses for the full panel. They are lower than those for one-way floors to allow for this non-uniform distribution of moments across the panel.



a) Fully banded tendons

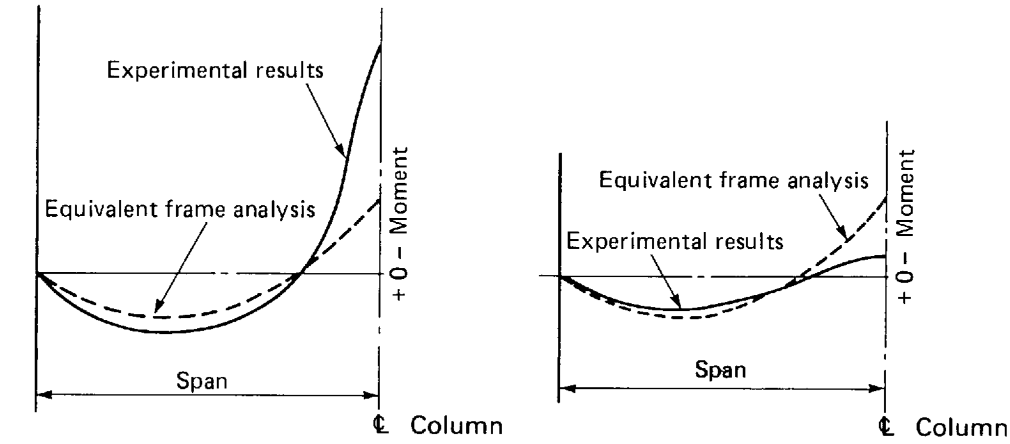


b) Uniformly distributed tendons



c) 50% banded plus 50% evenly distributed tendons over full width

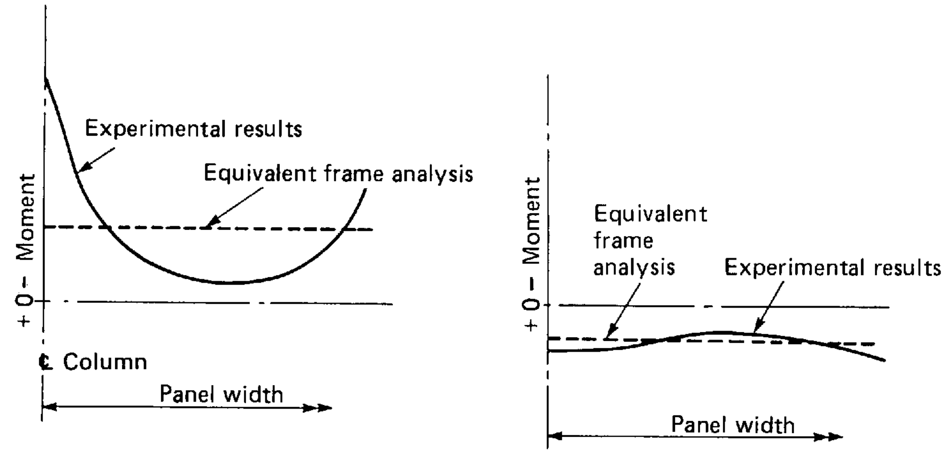
Figure 9: Bending moment surfaces and tendon diagrams for different tendon arrangements



a) Moments along section on column line

b) Moments halfway between column lines

Figure 10: Applied load bending moments in a solid flat slab.



a) Moments on column line

b) Moments halfway between column lines

Figure 11: Distribution of applied load bending moments across the width of a panel in a solid flat slab.

In contrast the sagging moments across the slab in mid-span regions are almost uniformly distributed across the panel width as shown in Figure 11b.

It is helpful to the understanding of post-tensioned flat slabs to forget the arbitrary column strip, middle strip and moment percentage tables which have long been familiar to the designer of reinforced concrete floors. Instead, the mechanics of the action of the tendons will be examined first.

The "load balancing" approach is an even more powerful tool for examining the behaviour of two-way spanning systems than it is for one-way spanning members. By the balanced load approach, attention is focused on the loads exerted on the floor by the tendons, perpendicular to the plane of the floor. As for one-way floors, this typically means a uniform load exerted upward along the major portion of the central length of a tendon span, and statically equivalent downward load exerted over the short length of reverse curvature. In order to apply an essentially uniform upward load over the entire floor panel these tendons should be uniformly distributed, and the downward loads from the tendons should react against another structural element. The additional element could be a beam or wall in the case of one-way floors, or columns in a two-way system. However, a look at a plan view of a flat slab (see Figure 12) reveals that columns provide an upward reaction for only a very small area. Thus, to maintain statical rationality, we must provide, perpendicular to the above tendons, a second set of tendons to provide an upward load to resist the downward load from the first set. Remembering that the downward load of the uniformly distributed tendons occurs over a relatively narrow width under the reverse curvatures and that the only available exterior reaction, the column, is also relatively narrow, it becomes obvious that the second set of tendons should be in narrow strips or bands passing over the columns.

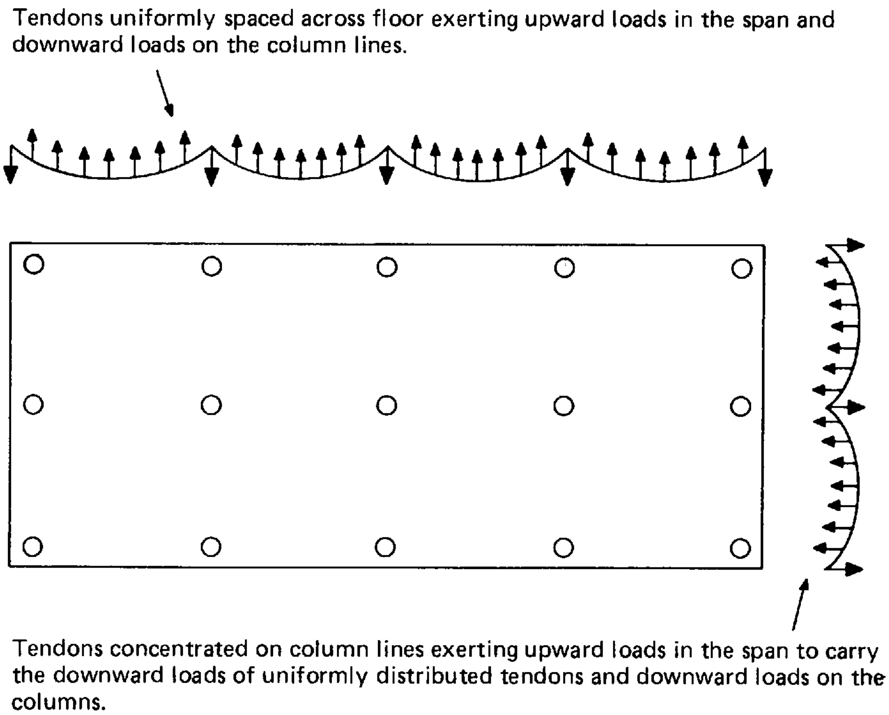


Figure 12: Load balancing with prestress tendons for regular column layouts.

There are two ways of accomplishing this two-part tendon system to obtain the nearly uniform upward load we desire for ease of analysis. In the first method, tendons are spaced uniformly in each of two directions and react against banded tendons along the column grid lines in each direction. This results in some of the tendons in each direction being banded over the columns, and some uniformly distributed between these bands (see Figure 13). This method works well where the columns are arranged on a rectangular grid.

Figure 9 shows the bending moments derived from grillage analysis of square panels with differing arrangement of tendons. The balanced load provided by the tendons in each direction is equal to the dead load. Figure 9c gives the most uniform distribution of moments and provides a practical layout of tendons. This arrangement gives 70% of the tendons in the banded zone and the remaining 30% between the bands. It should be noted that, since the width of the banded zone is 0.4 times the width of the bay, this arrangement is identical to providing 50% of the tendons evenly distributed over the full width of the bay in addition to 50% concentrated in the band. However, as can be seen from Figure 9 the detailed distribution is not critical provided that sufficient tendons pass through the column zone to give adaquate protection against punching shear and progressive collapse.

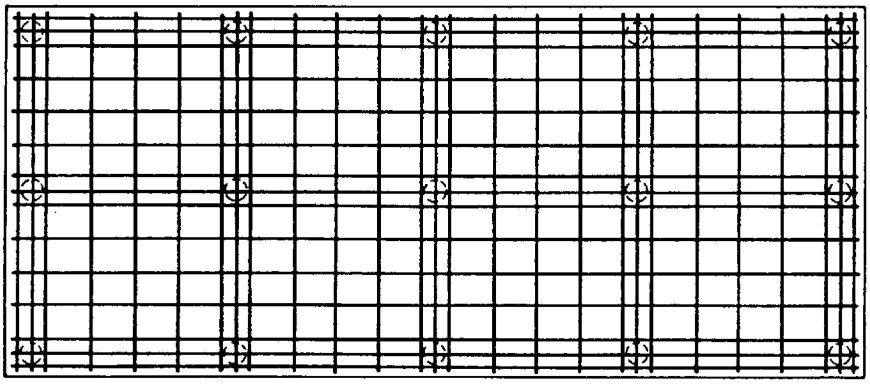


Figure 13: Tendons geometrically banded in each direction.

Where the column arrangement is irregular, or where openings or other geometric considerations require it, a second method may be used. In this method the uniformly distributed tendons and banded tendons may each be placed in one direction only (see Figure 14). The power of the second method becomes very clear by examining a floor which has columns of irregular layout. For an example, with reference to Figure 15a, it is assumed that the odd numbered grid lines are offset one half bay from the even numbered grid lines, but columns on a given letter grid are aligned.

If the 'column strip' approach illustrated in Figure 15b is retained as for conventionally reinforced floors, each span which started in a column strip ends in a middle strip, and tracing of load paths, a rational analysis, and proportioning reinforcement become difficult if not impossible and force a return to the basic idea of balancing loads with tendons. The uniformly distributed system of tendons (parallel to letter grid lines) can be accomplished with little regard for column location. It is only necessary to place the high points of the tendon profile (where reverse curvature and downward load occur) at the intersection of the tendons with the number grid lines. This system then reacts against the banded tendons placed on the number grid lines as shown in Figure 15c. By this procedure, the reaction of the gravity load balanced by the tendons is carried directly to columns, without any flexural action of the floor. Since this balanced load is typically a large portion of the permanent load on the floor, errors in analysis which are due to incorrect assumptions of load path are a function of relatively small loads, and thus are small. The possible consequences of such errors can be investigated by examining the behaviour of the floor under overloads.

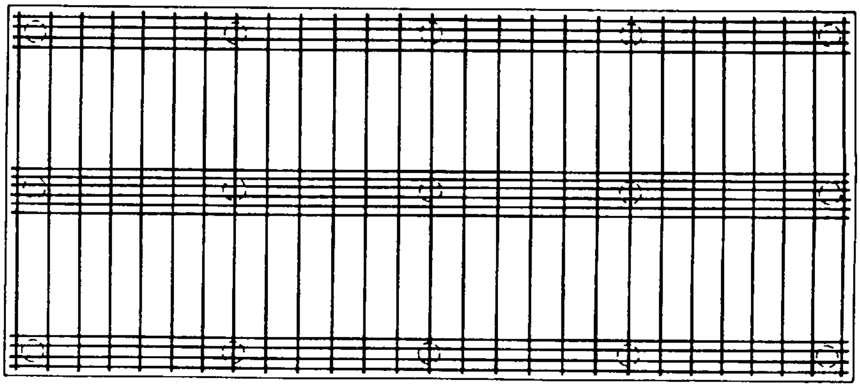


Figure 14: Tendons fully banded in one direction and uniformly distributed in the other direction.

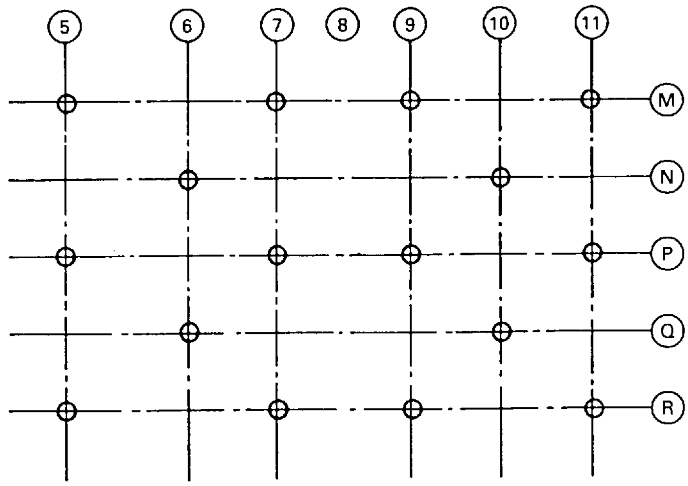
Flexural cracking is initiated at column faces and can occur at load levels in the serviceability range. While these and early radial cracks remain small, they are unlikely to affect the performance of the slab. Compression due to prestress delays the formation of cracks, but it is less efficient in controlling cracking than un-tensioned reinforcement placed in the top of floors, immediately adjacent to, and above the column.

2.4.1 *Flat slab criteria*

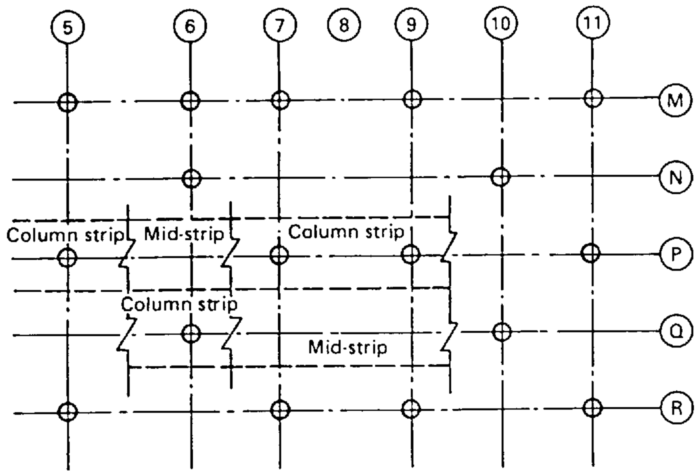
For a prestressed floor to be considered as a flat slab the following criteria apply:

- Precompression should be applied in two orthogonal directions:
  Such a floor with no, or moderate, crack formation performs as a homogeneous elastic plate with its inherent two-way behaviour. The actual tendon location at a given point in a floor system is not critical to the floor's two-way behaviour since precompression, which is the decisive factor, is commonly applied to the floor at its perimeter.

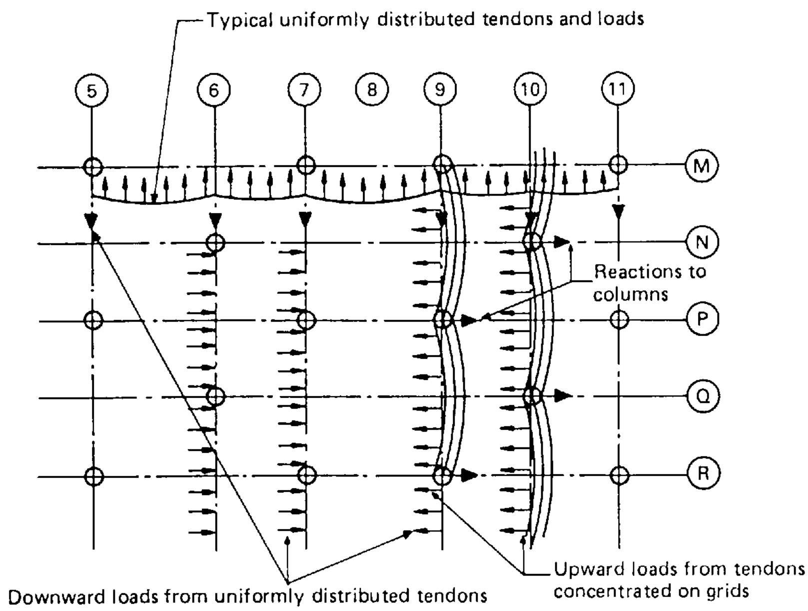
  The precompression at the edges of the slab is concentrated behind the anchorages, and spreads into the floor with increasing distance from the edge. This is true for floors of uniform thickness as well as floors with beams in the direction of precompression. Floors with banded post-tensioning and floors with wide shallow beams also qualify for two-way action at regions away from the free edges where precompression is attained in both directions.



a) Irregular flat plate column layout.



b) Column strips and middle strips



c) Load balancing with bonded tendons

Figure 15: Load balancing with prestressing tendons for irregular column layouts

Past experience shows that for the precompression to be effective it should be at least 0.7 N/mm^2 in each direction.

- Aspect ratio (length to width) of any panel should not be greater than 2.0: This applies to solid flat slabs, supported on orthogonal rows of columns. For aspect ratios greater than 2.0 the middle section will tend to act as a one-way spanning slab.

- Stiffness ratios in two directions:
  The ratio of the stiffness of the slab in two orthogonal directions should not be disproportionate. This is more likely to occur with non-uniform cross-sections such as ribs. For square panels this ratio should not exceed 10.0, otherwise the slab is more likely to behave as one-way spanning.

2.5 Shear

The method given in BS8110[4] for calculating shear in beams and one-way spanning slabs should be used. A method for calculating shear for post-tensioned flat slabs is not provided in BS8110. The method given in this manual (see 6.11.2) combines the prestress effects given in section 4 of BS8110 with the method given for punching shear for reinforced concrete in section 3 of BS8110.

3. STRUCTURAL FORM

3.1 Column layout

Current experience in many countries indicates a minimum span of approximately 7m to make prestressing viable in a floor. However, examples are known in which prestressed floors have been competitive where shorter spans have been used for architectural reasons, but prestressing was then only made viable by choosing the right slab form. In general the ideal situation is, of course, to 'think prestressing' from the initial concept of the building and to choose suitably longer spans.

In choosing column layouts and spans for a prestressed floor, several possibilities may be considered to optimise the design, which include:

a) Reduce the length of the end spans or, if the architectural considerations permit, inset the columns from the building perimeter to provide small cantilevers. Consequently, end span bending moments will be reduced and a more equable bending moment configuration obtained.

b) Reduce, if necessary, the stiffness of the columns to minimise the prestress lost in overcoming the restraint offered to floor shortening (see Section 3.3).

c) Where span lengths vary, adjust the tendon profiles and the number of tendons to provide the uplift required for each span. Generally this will be a similar percentage of the dead load for each span.

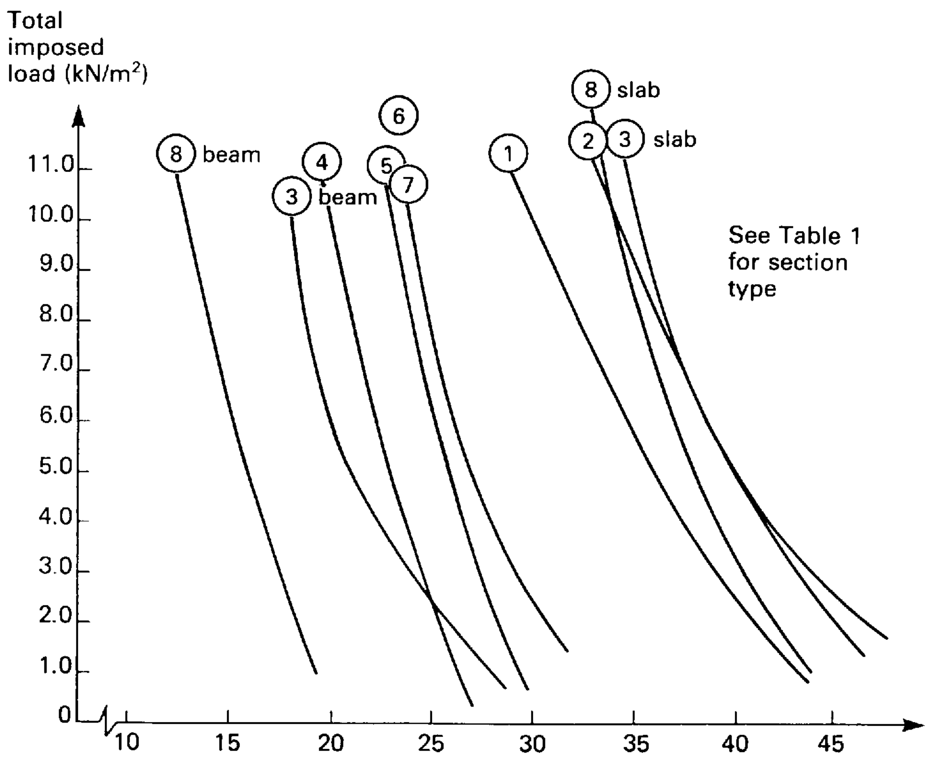
Once the column layout has been determined, the next consideration is the type of floor to be used. This again is determined by a number of factors such as span lengths, magnitude of loading, architectural form and use of the building, special requirements such as services, location of building, and the cost of materials available.

3.2 Floor thickness and types

The slab thickness must meet two primary functional requirements - structural strength and deflection. Vibration should also be considered where there are only a few panels. The selection of thickness or type (e.g. plate without drops, plate with drops, coffered or waffle, ribbed or even beam and slab) is also influenced by concrete strength and loading. There are likely to be several alternative solutions to the same problem and a preliminary costing exercise may be necessary in order to choose the most economical.

The information given in Figures 16, 17 and 18 will assist the designer to make a preliminary choice of floor section. Figure 16 (derived from Table 1) gives typical imposed load capacities for a variety of flat slabs and one-way floors over a range of span/depth ratios. These figures are based on past experience. Figure 16 is appropriate for all types of prestressed floor. Figures 17 and 18 are only appropriate for flat slabs but Figure 17 is not appropriate for coffered slabs which do not have a solid section over the column.

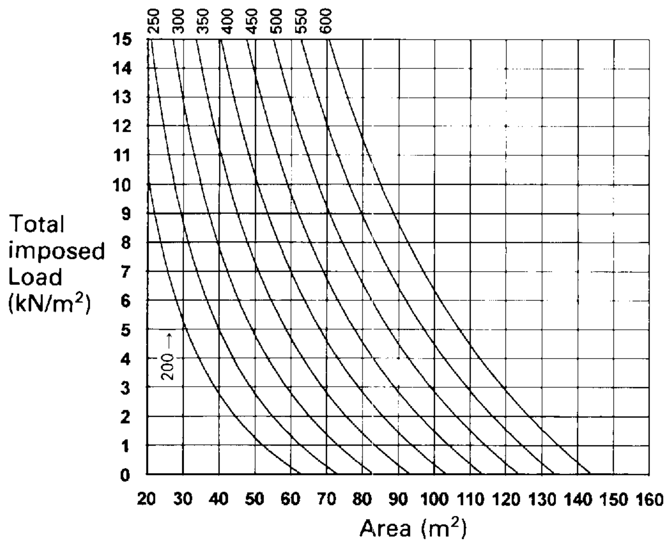
At this stage it should be noted that the superimposed load used in Figures 16, 17 and 18 consists of all loading (dead and live) bar the self-weight of the section. The calculation methods used for obtaining the graphs in Figures 17 and 18 are described in Appendix F.



Note:

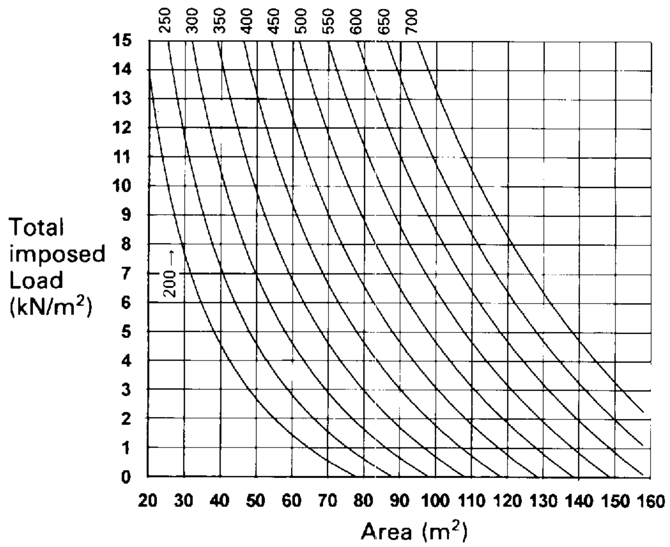
This chart is derived from the emperical values given in Table 1 for multi-span floors. For single-span floors the depth should be increased by approximately 15%.

Figure 16: Preliminary selection of floor thickness for multi-span floors.



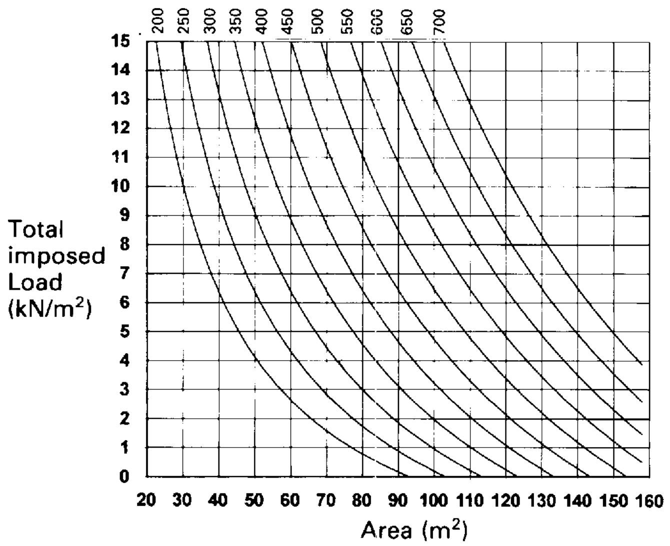
Slab thickness adjacent to columns

vc = 0.75 N/mm^2
Column size including head = 300 x 300 mm



Slab thickness adjacent to columns

vc = 0.75 N/mm^2
Column size including head = 500 x 500 mm



Slab thickness adjacent to columns

vc = 0.75 N/mm^2
Column size including head = 700 x 700 mm

Figure 17: Preliminary shear check for slab thickness at internal column.

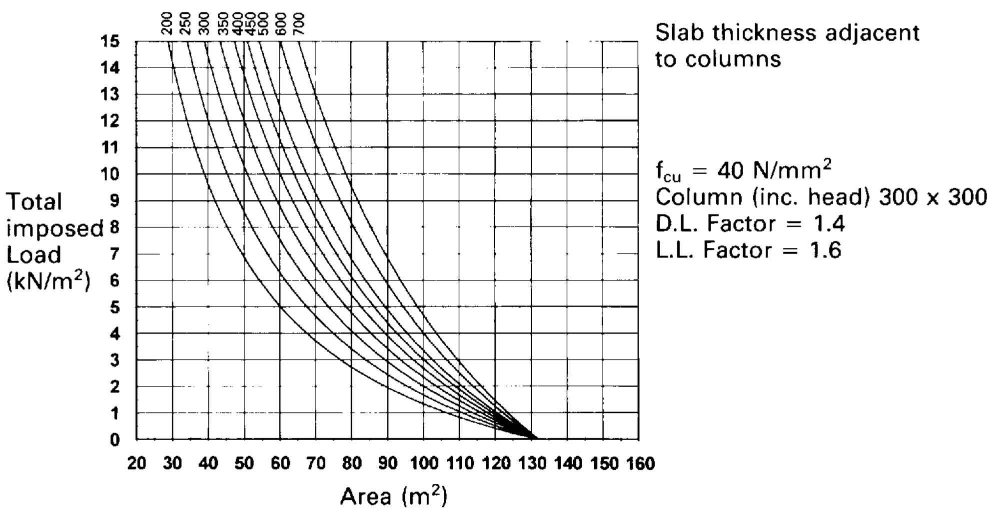


Figure 18: Ultimate shear check for flat slab at face of internal column.

Notes:

1. For column sizes other than 300 x 300 the slab depth should be multiplied by the factor (column perimeter / 1200).
2. The maximum shear stress for f_{cu} = 40 N/mm^2 and more is 5N/mm^2.
   For f_{cu} < 40 N/mm^2 the maximum shear stress is $0.8\sqrt{f_{cu}}$.
   For f_{cu} = 35 N/mm^2 increase slab depth by a factor of 1.06.
   For f_{cu} = 30 N/mm^2 increase slab depth by a factor of 1.14.
3. The value of d/h is assumed to be 0.85.
4. The ratio of V_{eff}/V is assumed to be 1.15.
5. These curves do not take account of elastic distribution effects. See Section 6.6.

Flat slabs tend to exceed punching shear limits around columns, and often need additional shear reinforcement at these locations. The graphs in Figure 17 provide a preliminary assessment as to whether shear reinforcement is needed for the section types 1, 2, 3, 5 and 6 (all flat slabs) in Table 1. As the shear capacity of a slab is dependent on the dimensions of the supporting columns or column heads, each graph has been derived using different column dimensions.

In addition, the shear capacity at the face of the column should be checked. This can be done using the graph in Figure 18. The graph has been derived for slabs with 300 × 300mm supporting columns, and to obtain the imposed load capacities for slabs with other supporting column sizes, the values in the graph should be multiplied by the ratio of required column perimeter/1200.

The following procedure should be followed when using Table 1, Figures 16, 17 and 18 to obtain a slab section.

a) Knowing the span and imposed loading requirements, Figure 16 or Table 1 can be used to choose a suitable span/depth ratio for the section type being considered. Table 1 also provides a simple check for vibration effects.

b) If section type 1, 2, 3, 5, or 6 has been chosen, check the shear capacity of the section, using one of the graphs in Figure 17 (depending on what size of column has been decided upon). Obtain the imposed load capacity for the chosen slab section. If this exceeds the imposed load, then shear reinforcement is unlikely to be necessary. If it does not, then reinforcement will be required. If the difference is very large, then an increase in section depth or column size should be considered.

c) Check the shear capacity at the face of the column using the graph in Figure 18. If the imposed load capacity is exceeded, increase the slab depth and check again.

It should be noted that Table 1 and Figure 16 are applicable for multi-span floors only. For single-span floors the depth should be increased by approximately 15%. Figures 17 and 18 are applicable for both floor types and have been derived using an average load factor of 1.5 (see Appendix F).

Figures 17 and 18 are set for internal columns. They may be used for external columns provided that the loaded area is doubled for edge and quadrupled for corner columns. This assumes that the edge of the slab extends to at least the centre line of the column.

Table 1: Typical span/depth ratios for a variety of section types for multi-span floors.

* Additional requirements if no vibration check to be carried out for normal office conditions:

A either the floor has at least four panels and is at least 250 mm thick or the floor has at least eight panels and is at least 200 mm thick.

B either the floor has at least four panels and is at least 400 mm thick or the floor has at least eight panels and is at least 300 mm thick

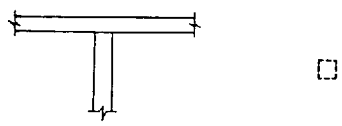
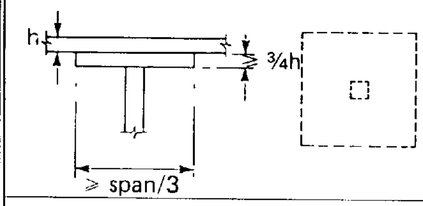
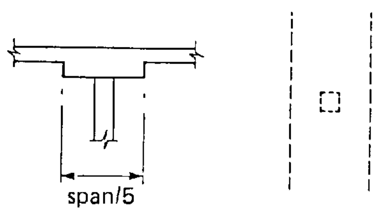
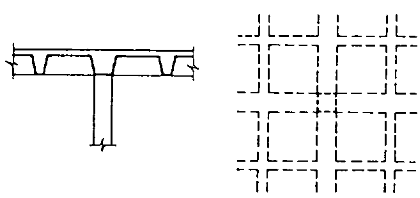
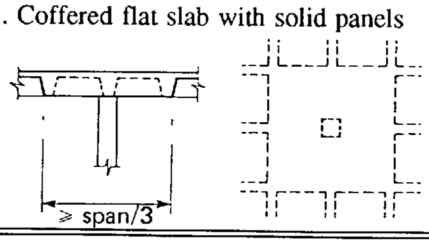
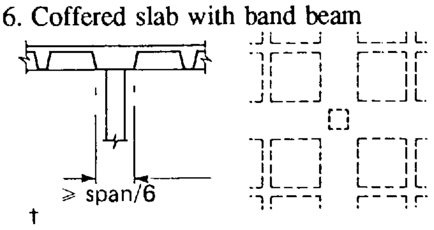
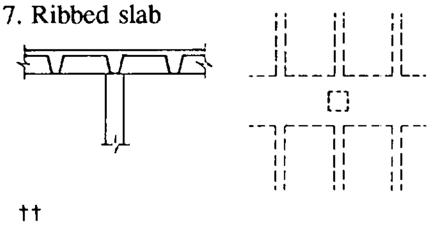
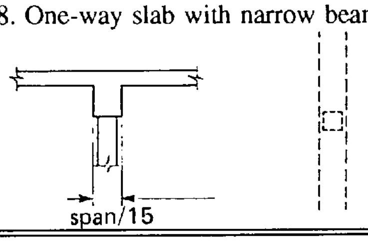
Section type	Total imposed loading (kN/m^2)	Span/depth ratios $6\ m \leq L \leq 13\ m$		*
1. Solid flat slab 	2.5	40		
	5.0	36		A
	10.0	30		
2. Solid flat slab with drop panel 	2.5	44		
	5.0	40		A
	10.0	34		
3. Banded flat slab 		slab	beam	
	2.5	45	25	
	5.0	40	22	A
	10.0	35	18	
4. Coffered flat slab 	2.5	25		
	5.0	23		B
	10.0	20		
5. Coffered flat slab with solid panels 	2.5	28		
	5.0	26		B
	10.0	23		

Table 1. *Continued*

Section type	Total imposed loading (kN/m²)	Span/depth ratios $6\ m \leq L \leq 13\ m$		*
6. Coffered slab with band beam 	2.5 5.0 10.0	28 26 23		B
7. Ribbed slab 	2.5 5.0 10.0	30 27 24		B
8. One-way slab with narrow beam 	2.5 5.0 10.0	slab 42 38 34	beam 18 16 13	A

Notes:

1. All panels assumed to be square
2. Span/depth ratios not affected by column head
3. † It may be possible that prestressed tendons will only be required in the banded sections and that untensioned reinforcement will suffice in the ribs, or vice versa.

   †† The values of span/depth ratio can vary according to the width of the beam.

3.3 *Effect of restraint to floor shortening*

Post-tensioned floors must be allowed to shorten to enable the prestress to be applied to the floor[8, 9]. Shortening occurs because of:

a) Elastic shortening due to the prestress force.

b) Creep shortening due to the prestress force.

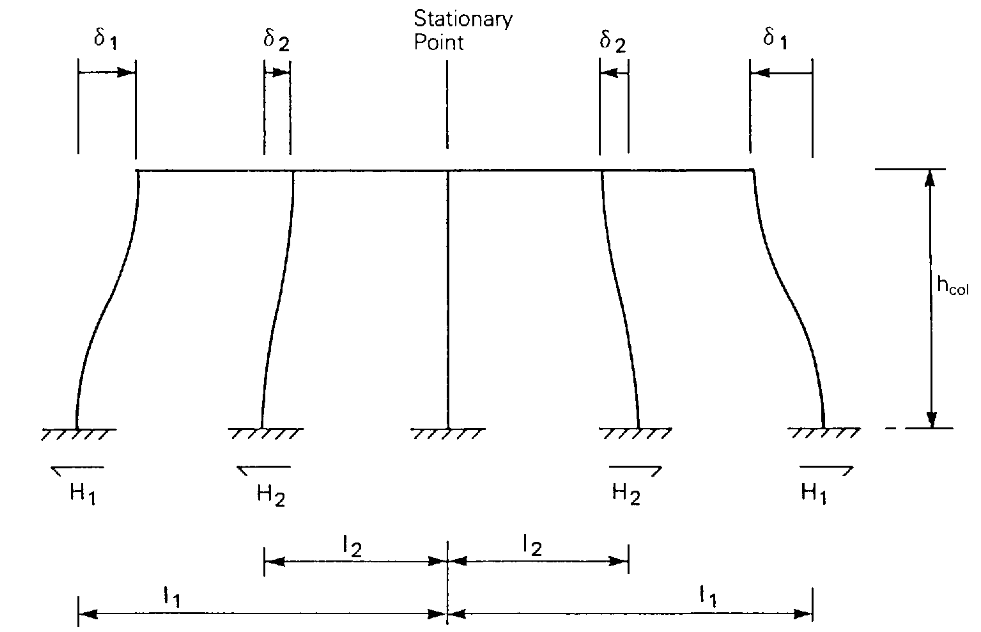
c) Shrinkage of concrete.

The elastic shortening occurs during stressing of the tendons, but the creep and shrinkage are long-term effects.

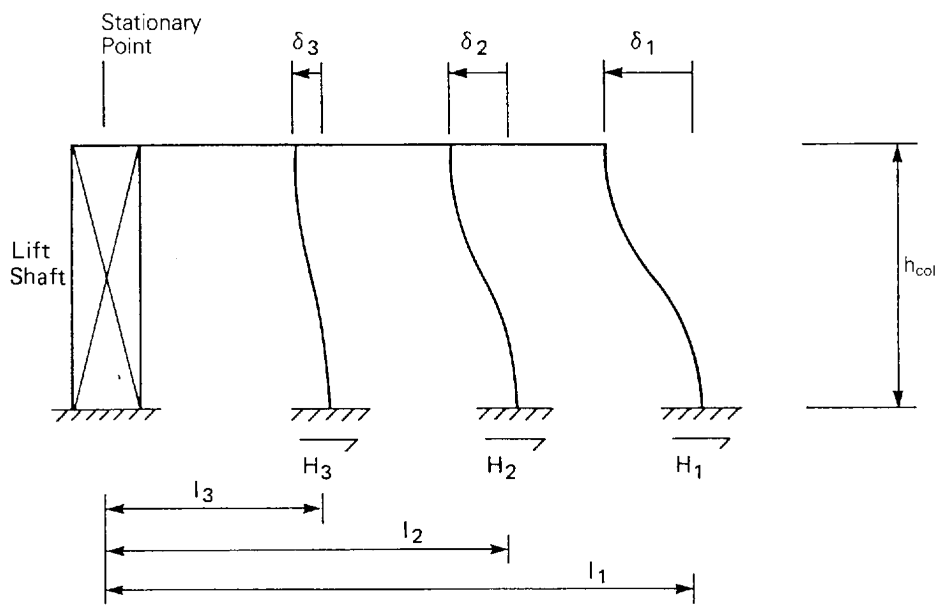
The floor will be supported on columns or a combination of columns and core walls. These supports offer a restraint to the shortening of the floor. There are no firm rules which may be used to determine when such restraint is significant. As a guide, if the prestress is less than 2N/mm², the floor is not very long and there is not more than one stiff restraint (i.e. a lift shaft) then the effects of restraint are usually ignored.

A simple method of ascertaining the restraint offered by the supports is to calculate the elastic, creep and shrinkage strains expected in the slab and then to calculate the

forces required to deflect the supports. Figure 19 shows two simple frames in which the floors have shortened and the columns have been forced to deflect. The force in each column may be calculated from the amount it has been forced to deflect and its stiffness. The stiffness may be calculated on the assumption that the column is built-in at both ends.



(a) Symmetrical floor supported on columns



(b) Floor supported by columns and lift shaft at one end

Figure 19: Restraint to floor shortening.

The calculation of elastic, creep and shrinkage strains may be based on the values given in BS8110[4]. The elastic strain should be based on the modulus of elasticity at the time the tendons are stressed. If this is at seven days after casting the modulus is approximately 80% of the modulus at 28 days. The creep strain depends on the age of the concrete when the tendons are stressed, the humidity and the effective thickness. The creep strain would be typically 2.5 times the elastic strain. The shrinkage strain will generally be in the range 100 to 300 x 10^{-6}, but in some circumstances it can increase to 400 x 10^{-6}.

Typical strains for a 300mm internal floor with a prestress of 2 N/mm^2 would be:

Elastic strain	-	100 x 10^{-6}
Creep strain	-	250 x 10^{-6}
Shrinkage strain	-	300 x 10^{-6}
Total long-term strain (ε_{LT})	-	650 x 10^{-6}

The following analysis is approximate but conservative and ignores any displacement of the foot of the columns or rotation of the ends of the columns. A more accurate analysis may be made using a plane frame with imposed member strains.

The force required to deflect each column, as shown in Figure 19, may be assumed to be calculated as follows:

$$\delta_i = \varepsilon_{LT} \times l_i$$

$$H_i = \frac{12E_cI_i\delta_i}{(h_{col})^3}$$

For the purposes of calculating H_i, the value of E_cI_i for the column may be reduced by creep in the column and in some cases cracking. A reduction of at least 50% from the short-term elastic properties is normally justifiable.

The total tension in the floor due to the restraint to shortening is the sum of all the column forces to one side of the stationary point. In Figure 19a, the tension is $H_1 + H_2$; in Figure 19b, the tension is $H_1 + H_2 + H_3$. This tension acts as a reduction in the precompression of the floor by the prestress. If the tension is small in comparison with the prestress, it may be ignored. If the tension force is moderate, it may be necessary to subtract it from the prestress to obtain the effective precompression of the floor. But if the restraint is so severe that flexing of the vertical members to accommodate the shortening is not possible, other measures are required. These may include freeing the offending stiff elements during a temporary condition. However, it must be remembered that creep and shrinkage will continue to occur for up to 30 years.

4. MATERIALS

4.1 Concrete

Concrete should be mixed, transported and placed in accordance with BS8110, Part 1, Section 6[4]. Choice of concrete type and grade will be influenced by durability, early strength gain requirements, material availability and basic economics. At present concrete grades of C35 and C40 are the most commonly used for post-tensioned floors.

Where lightweight aggregates are used, references should be made to the special requirements of BS8110, Part 2, Section 5[4].

4.2 Tendons

4.2.1 Strand

The tendon material used for post-tensioning concrete floors is normally 7-wire strand. This strand should comply with Type 2 (low relaxation) as described in BS 5896, Table 6[10].

4.2.2 Tendon protection

Unbonded tendons

Unbonded tendons are protected by a layer of grease inside a plastic sheath. An example is shown in Figure 20. These materials should comply with the recommendations given in reference 11.

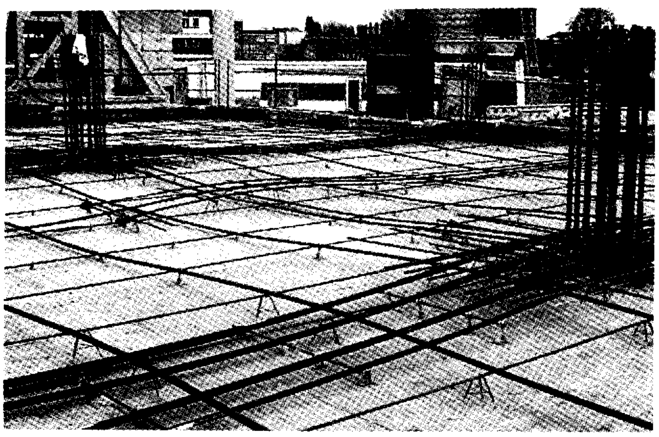


Figure 20: Layout of unbonded tendons.

Under normal conditions, the strand is supplied direct from the manufacturer already greased and sheathed. In no circumstances should PVC be used for the plastic sheath, as it is suspected that chloride ions can be released in certain conditions.

Bonded tendons

Bonded tendons are placed in metal ducts which can be either circular or oval in form. An example is shown in Figure 21. The latter is used in conjunction with an anchorage which ensures that up to four strands are retained in the same plane in order to achieve maximum eccentricity.

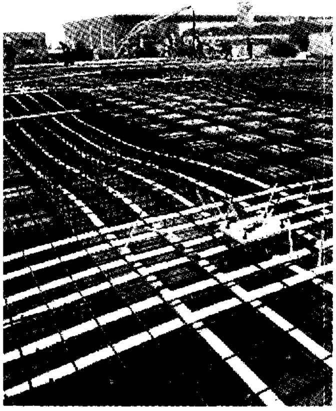


Figure 21: Layout of bonded tendons.

The ducts are made from either spirally wound or seam folded galvanised metal strip. On completion of stressing, the ducts are pumped full of cement grout which effectively bonds the strand to the structure as well as ensuring corrosion protection. Further information can be obtained from reference 12.

4.2.3 *Anchorages*

Anchorage components should comply with BS 4447[13]. Details of these are shown in Figures 22 and 23. In the case of unbonded anchorages corrosion protection should comply with Class A exposure as defined in reference 14. In addition, tests for unbonded anchorages should include fatigue testing consisting of cycling the prestressing force between 60 and 65% of the characteristic strength of the strand for two million cycles.

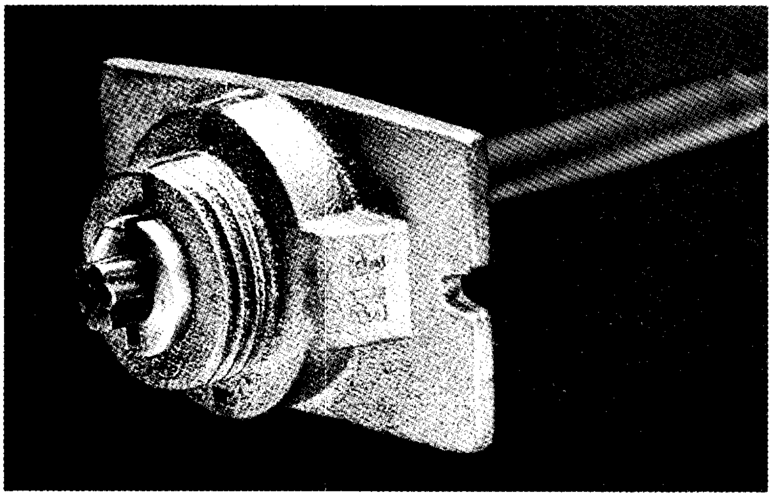


Figure 22: A typical anchorage for an unbonded tendon.

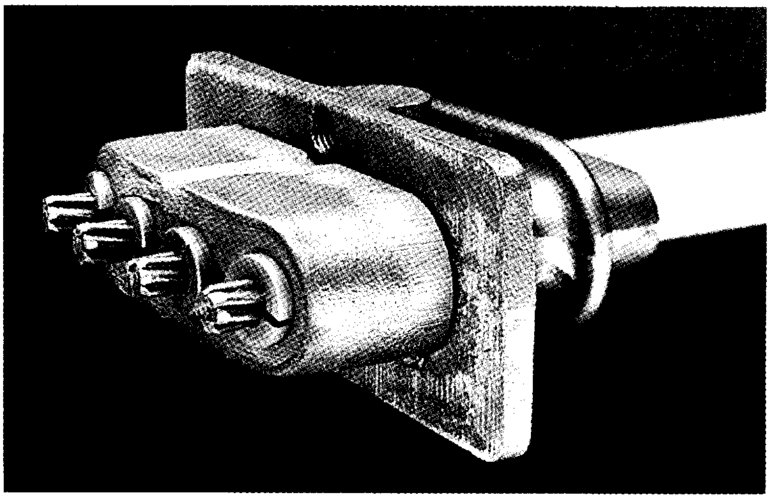


Figure 23: A typical anchorage for a bonded tendon.

4.3 Un-tensioned reinforcement

Un-tensioned reinforcement shall comply with BS 4449[15].

5. COVER REQUIREMENTS

Nominal cover is dependent on durability requirements or fire resistance, whichever condition is the more onerous.

Bonded tendons: The cover to the tendons should be in accordance with the requirements for prestressed concrete in BS8110, Part 1, Clause 4.12.3[4] the cover being measured to the outside of the duct. It should be noted that the cover to the centre of the tendon will be more than that to the centre of the duct, since the tendon will press against the wall of the duct.

Unbonded tendons: There is no durability requirement for unbonded tendons protected in accordance with 4.2.2. Fire protection shall be provided in accordance with BS8110, Part 1, Clause 4.12.3.1.3[4] and the nominal cover to the sheath should not be less than 25mm. The tendon is normally specified as a nominal diameter (e.g. 12.9 or 15.7mm for 7-wire super strand): 3mm should be added to the diameter to allow for the thickness of sheathing.

Un-tensioned reinforcement: The cover to the un-tensioned reinforcement should be in accordance with the requirements for reinforced concrete in BS8110, Part 1, Clause 3.3[4].

Anchorages: The cover to anchorages should be as for bonded tendons given in BS8110, Part 1, Clause 4.12.3.1[4].

Consideration should be given to the layout of both tendons and un-tensioned reinforcement when deciding the critical cover requirements (see Section 7.5).

6. THE DESIGN PROCESS

6.1 Introduction

This section considers the various stages of the design process in more detail. As in most reinforced and prestressed concrete design work, the customary design process is of an iterative nature following the cycle:

1. Preliminary design
2. Check design by analysis
3. Revise design as required
4. Repeat steps 2 and 3 if necessary.

The analysis is normally based on semi-empirical procedures such as the equivalent frame method. More rigorous analyses based on, for example, finite element methods are rarely adopted. They should only be considered for large projects of unusual form where the high design costs and the inapplicability of the empirical method justify them.

The design is undertaken generally in accordance with BS8110[4] with additional guidance given in this report. Normally the flexural capacity at Serviceability Limit State is considered first, and then checks on flexural and shear capacity at Ultimate Limit State are carried out.

6.2 Design flow chart

A typical design flow chart is shown in Figure 24.

6.3 Basic analysis

The analysis of post-tensioned floor systems differs from a reinforced concrete design approach owing to the positive effect that the tendons have on the structure. In reinforced concrete the reinforcement is initially unstressed; the stress in the reinforcement results from the deformation and cracking of the structure under applied load. In this way the reinforcement may be considered to act passively. On the other hand, the tendons in a post-tensioned floor are actively stressed by the jacks so that they are loaded before the application of other loads. The force in the tendon is chosen by the designer and does not vary much with the application of Serviceability Limit State dead and live loads.

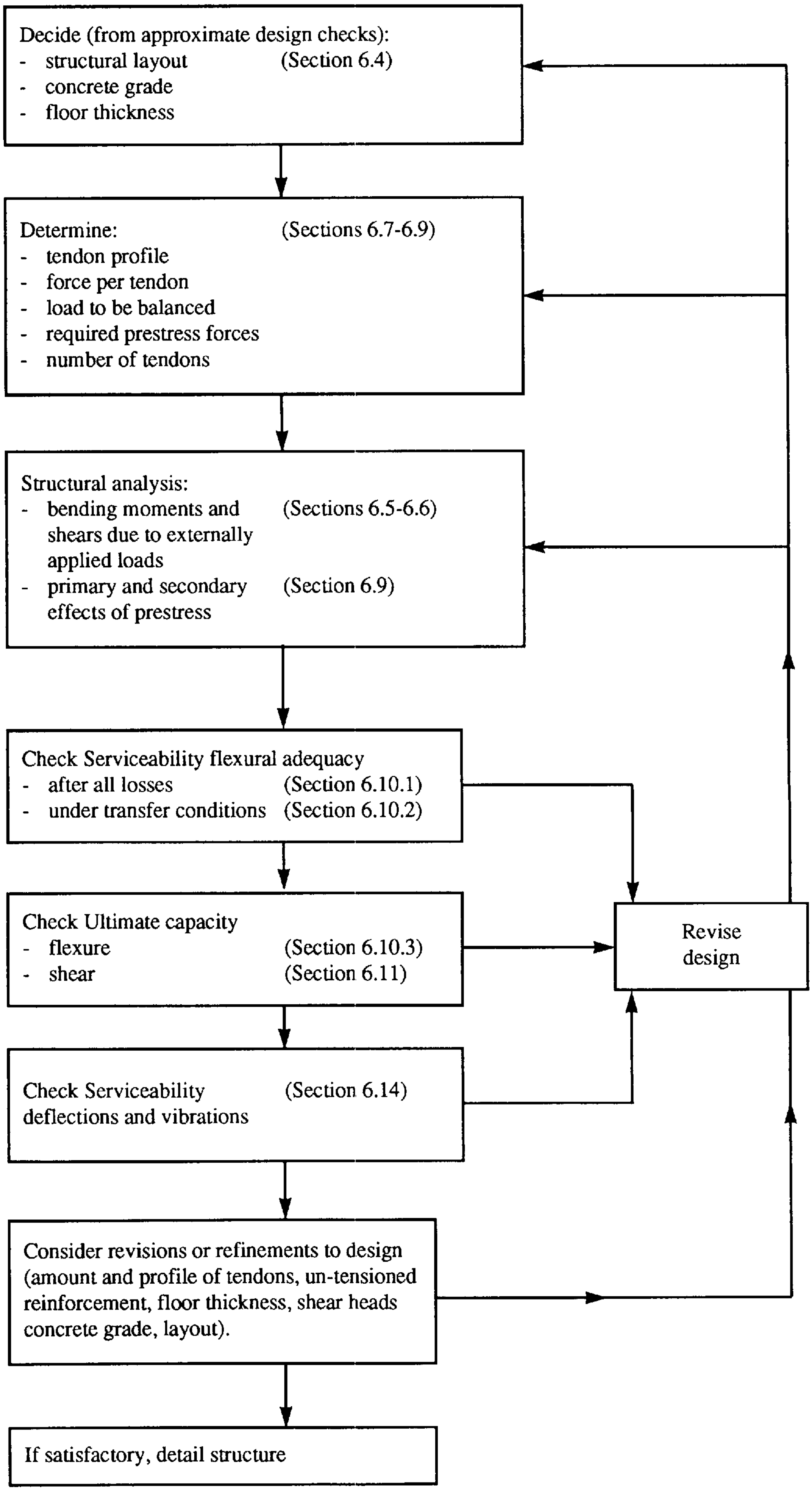


Figure 24: Design flow chart

The analysis of equivalent frames may be undertaken by hand, using moment distribution or flexibility methods, or by computer using plane-frame analysis programs. There are also available on the market several computer programs specially written for post-tensioned flooring systems. These programs not only undertake the analysis of the frame under applied loading and loading from the tendons, but also calculate the flexural stresses. For more complex or detailed analysis, grillage or finite element methods may be used. Whichever technique is used for the structural analysis it must take into account not only the dead and live loads but also the loads which the tendons apply to the structure (see Section 6.7).

6.4 *Structural layout*

The choice of layout and member sizing has been discussed in Section 3, and is probably the most important decision in the design process. Unless previous experience or overriding factors dictate the exact form and section, several possibilities should be studied, although the designer should be able to limit the possible solutions by considering the various constraints and by rough design and costing exercises.

With regard to slab thickness and concrete strengths, the relationship of structural layout, slab thickness and loading has been referred to in Section 3. A determination of a trial member depth must be made at an early stage in the calculation process. This can often be best obtained by assuming a value of about 70% of the equivalent non-prestressed member.

6.5 *Loading*

The loading for Serviceability Limit State should consider the dead load and post-tensioning effects acting with those combinations of live loads which result in the maximum stresses. Unless there are specific abnormal loads present, it will generally be sufficient to consider the post-tensioning effects in combination with the live loads as given in BS8110, Part 1, Clause 4.3.3[4].

At transfer of prestressing only the dead loads present during stressing, together with the post-tensioning effects before losses due to creep, shrinkage and relaxation, should be considered in obtaining stresses. Where the applied loads change significantly during construction or phased stressing is employed, the various stages should each be checked for transfer stress limits.

At the Ultimate Limit State the load combinations shown in BS8110, Part 1, Table 2.1 and Clause 4.3.3[4] shall be considered to arrive at the maximum moments and shears at any section. Secondary effects of prestressing should be included in the applied loads with a load factor of 1.0 (see Section 6.9).

6.6 *Equivalent frame analysis*

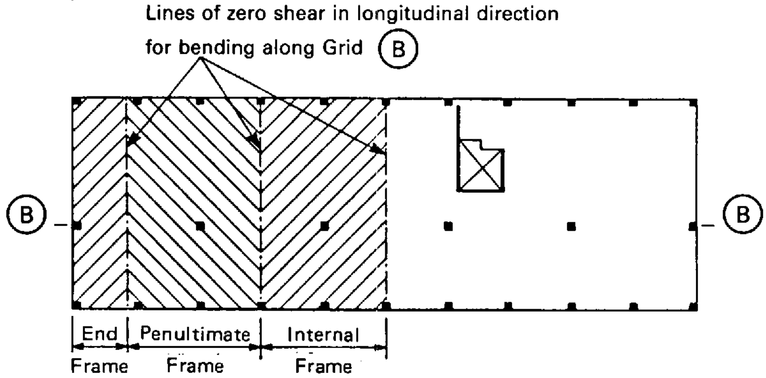
It is usual to divide the structure into sub-frame elements in each direction. Each frame usually comprises one line of columns together with beam/slab elements of one bay width. The frames chosen for analysis should cover all the element types of the complete structure.

The ends of the columns remote from the sub-frame may generally be assumed to be fixed unless the assumption of a pinned end is clearly more reasonable (e.g. pad footings).

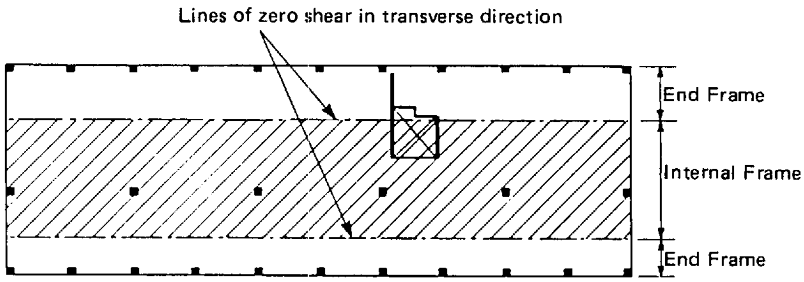
The use of the equivalent frame method does not take account of the two-dimensional elastic load distribution effects automatically. It will give different support reactions from the analyses in the two orthogonal directions unless the width of slab chosen coincides with the points of zero shear in the other direction. Normally for internal bays the width of slab will be the full panel width. However for a regular layout, the penultimate frame will pick up more than half the width on the side of the end bay (see Figure 25). Provided the reaction on each column is taken as the larger value from the two analyses little accuracy will be lost. However where the size and arrangement of edge columns is different from the internal columns the width of slab should be estimated more accurately. This will ensure the correct selection of the number of prestress tendons with the profile appropriate for the frame being analysed.

It should be noted that these elastic effects are automatically taken into account when the floor is analysed using grillage or finite element methods.

Irrespective of which analytical technique is used, care should be taken to ensure that the assumptions made are appropriate to the structure under consideration. In particular the prestress applied to two adjacent frames should not be very dissimilar otherwise the prestress from the more highly stressed frame will dissipate into the adjacent frames.



(a) Equivalent frame widths in transverse direction



(b) Equivalent frame widths in longitudinal direction

Figure 25: Elastic load distribution effects

BS8110, Part 1, Section 3.7.2[4] gives a clear definition on the division of a flat slab into sub-frames or 'panels'. Other methods may also be used.

It is now common to analyse structures using plane frame computer programs. However, when longhand moment-distribution calculations are employed, stiffness, carry-over factors and fixed end moment coefficients must be calculated. These can be quite complicated for varying sections, column heads and drop-panels and, although often ignored in hand calculations, the effect on stiffness of the complete beam moment of inertia over the column width can be most significant, particularly for wide columns.

It should also be noted that BS8110, Part 1, Section 3.7.2.6[4] allows reduction of negative moments to the column face, which equally applies to post-tensioned members.

6.7 Tendon profile and balanced load

Ideally the tendon profile is one which will produce a bending moment diagram of similar shape, but opposite sign, to the moments from the applied loads. This is not always possible because of varying loading conditions and geometric limitations (see Section 5).

It should be noted that for bonded systems the centroid of the strands will not coincide with the centroid of the duct. This is particularly true in the case of circular ducts. Further information may be available from the manufacturer's literature.

In the simplest case, for a uniformly loaded simply-supported beam, the bending moment is parabolic, as is the ideal tendon profile. The total 'sag' in the parabola is referred to as the tendon 'drape' (see Figure 26), and is limited by the section depth and minimum cover to the tendon. At the supports the tendon has no eccentricity and hence there is no bending moment due to the tendon forces.

Tendon profiles are not always symmetric. However, the point of maximum drape is still at the centre of the points of inflection, but may not correspond to the point of maximum sag.

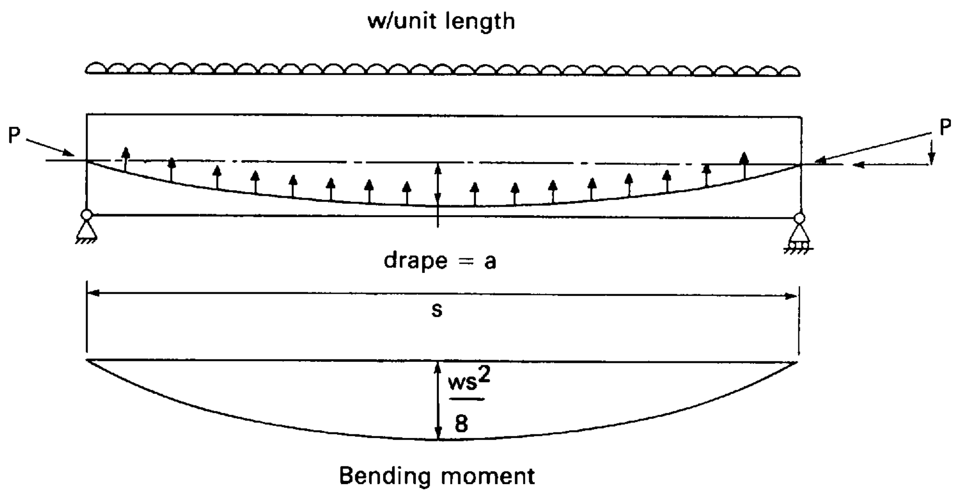


Figure 26: Idealised tendon profile.

The upward forces applied to the concrete by a parabolic profiled tendon, as shown in Figure 26, are uniformly distributed along the tendon. At the ends of the tendon downward forces are applied to the concrete by the anchorages. The upward and downward forces are in equilibrium so that no external forces occur. The set of forces applied to the member by the tendon are known as the 'equivalent' or 'balanced' loads, in that the upward forces counter-balance a proportion of the downward forces due to dead and live loads.

For a parabolic profile the upward uniformly distributed load, w, can be calculated as follows:

$$\frac{ws^2}{8} = P_{av}a$$

or

$$w = \frac{8aP_{av}}{s^2}$$

where:

s	=	distance between points of inflection
a	=	drape of tendon measured at centre of profile between points of inflection. Note that this may not be position of maximum sag
P_{av}	=	average prestressing force in tendon

Usually, in continuous members, the most effective use of a tendon in producing 'balanced loads' is achieved by having the tendon at its lowest possible point in positive moment locations, and at its highest possible point in negative moment locations. In this way the drape, and consequently the 'balanced loads', is increased to a maximum.

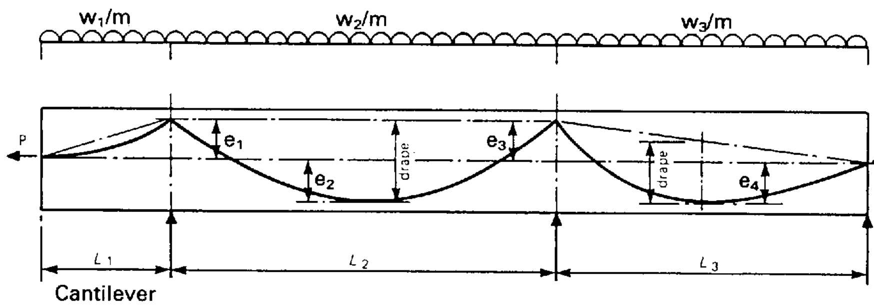
The 'equivalent' or 'balanced' loads may be applied to the structural frame in order to obtain the total effects of prestressing. The total effects are a combination of the Primary and Secondary effects as described in Section 6.9.

It is beyond the scope of this publication to give an extensive treatise on prestressing theory or load-balancing design. Further details may be obtained from reference 16.

In post-tensioned design it is common to roughly 'balance' equal proportions of the dead and applied loads in each span. Some designers set out with a preconceived idea of what load they wish to balance as a proportion of the dead or total load. Others balance the minimum amount which will result in the final stresses due to the out-of-balance loads being as close as possible to the maximum allowable stresses.

This latter approach is usually the most economical overall but may not always be the most suitable for deflection or congestion of un-tensioned reinforcement.

Figure 27 illustrates an idealised tendon profile for a two-span member with a cantilever. The parabolic profiles result in the balanced loads w_1, w_2 and w_3 as shown, calculated from the tendon profile and hence the 'drapes'.



Cantilever
Span 1: $Pe_1 = \frac{w_1 L_1^2}{2}$

Span 2: $P\left(\frac{e_1 + e_3}{2} + e_2\right) = \frac{w_3 L_3^2}{8}$

Span 3: $P\left(\frac{e_3}{2} + e_4\right) = \frac{w_3 L_3^2}{8}$

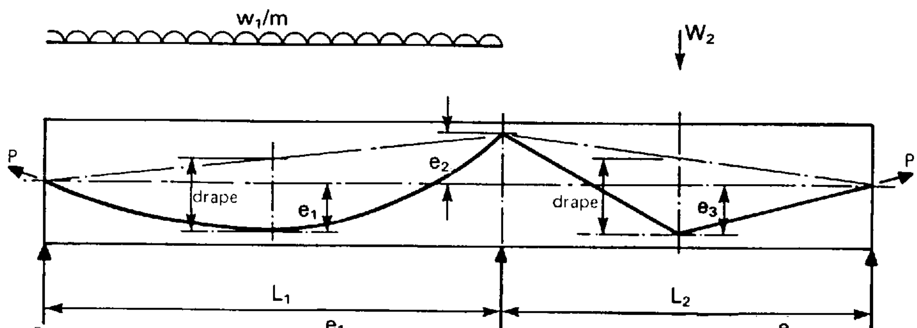
Figure 27: Idealised tendon profile for two spans with single cantilever.

Figure 28 illustrates a two-span member with an idealised tendon profile to provide a uniform uplift over span 1 and a concentrated uplift in span 2. The concentrated effect is useful in members transferring column or similar point loads.

While the bending moments 'peak' over the supports, it is clear that in practice a tendon cannot do this and some approximation must be made. Remember that the peak is where the tendon is 'dumping' the load it has picked up by its parabolic shape (Figure 29). In practice, tendon profiles are of the form shown in Figure 30.

The ratio L'/L should generally be kept as small as possible and is usually selected as 0.10. Appendix C provides information from which the parabolic tendon geometry can be calculated.

The resultant balancing forces are therefore as shown in Figure 31.



Span 1: Total drape = $e_1 + \frac{e_1}{2}$ Span 2: Total drape = $e_3 + \frac{e_2}{2}$

Balanced load $w_1 = P\, e_1$ total drape x $8/L_1^2$ Balanced load w_2 = P x total drape x $4/L_2$

P is the prestressing force at the section under consideration.

Note:

that the centre of gravity of the concrete and the centre of gravity of the tendon coincide at the end of the member so that no equivalent load moments are applied at the end of the member.

Figure 28: Idealised tendon profile for two spans with point load

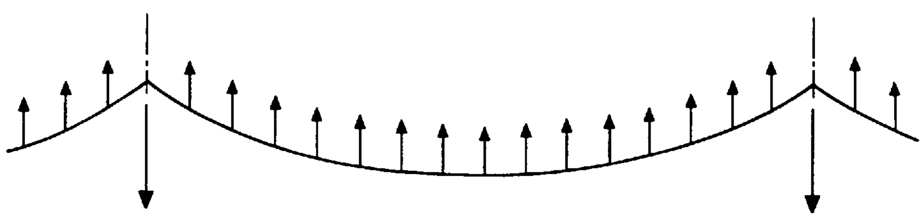


Figure 29: Load 'dumping' at 'peaks'.

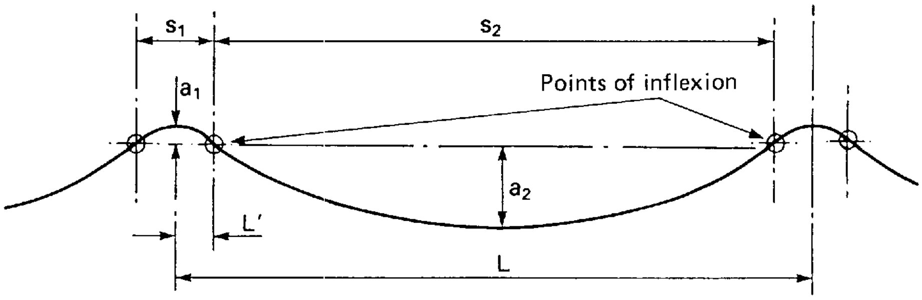


Figure 30: Practical representation of idealised tendon profile.

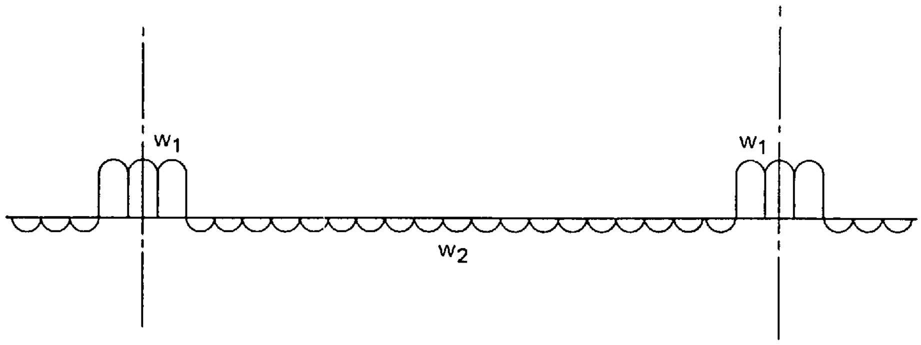


Figure 31: Resultant balancing forces.

For the reverse parabola at the support the total load downwards:

$$W_1 = w_1 \times s_1 = \frac{8Pa_1}{s_1}$$

and for the span parabola the total load upwards:

$$W_2 = w_2 \times s_2 = \frac{8Pa_2}{s_2}$$

If we make L'/L equal to 0.1, as suggested above,

then: $s_2 = 4s_1$

Since the upward and downward loads must be equal, it follows that:

$$\frac{a_1}{s_1} = \frac{a_2}{s_2}$$

and hence $a_1 = \dfrac{a_2}{4}$

The balancing loads upwards and downwards due to the tendons can thus be calculated.

6.8 *Prestress forces and losses*

From the time that a post-tensioning tendon is stressed, to its final state many years after stressing, various losses take place which reduce the tension in the tendon. These losses are grouped into two categories, namely:

1. Short-term Losses, which include:

   a) Friction losses in the tendon
   b) Wedge set or 'draw-in'
   c) Elastic shortening of the structure.

These losses take place during stressing and anchoring of the tendon.

2. Long-term Losses, which include:

   a) Shrinkage of the concrete
   b) Creep of the concrete under the effect of the prestress
   c) Relaxation of the steel tendon.

Although these losses occur over a period of up to ten or more years, the bulk occurs in the first two years following stressing. Typically, losses reduce the applied prestress force by approximately 10% at transfer and 20% after all losses. The calculation of losses is discussed in more detail in Appendix B.

6.9 *Secondary effects*

The secondary effects of prestressing are sometimes called 'parasitic effects' but that implies that the effects are unwanted and harmful. This is not in fact the case. For most structures the secondary moment will be a sagging moment and will increase the moments due to applied loads at midspan but reduce the moments at the support. In some structures it is possible to 'tune' the secondary effects by adjusting the shape of the tendon profile to obtain the optimum solution. This is more likely to be of use in the design of beams rather than slabs.

Primary prestressing forces and moments are the direct result of the prestress force acting at an eccentricity from the section centroid. The primary moment at a section is simply the sum of the products of each tendon force with its eccentricity; the primary shear is the sum of transverse components of the tendon forces and the primary axial load is the sum of the axial components of the tendon forces.

When an element of a structure is prestressed its shape changes. It will always shorten, and will bend if the centroid of the prestress force does not coincide at all positions with the section centroid. (It is possible, however, to select a tendon profile which results in no rotation of the element ends.)

If the element is part of a statically determinate structure then these changes in shape will not affect the distribution of forces and moments (Figure 32).

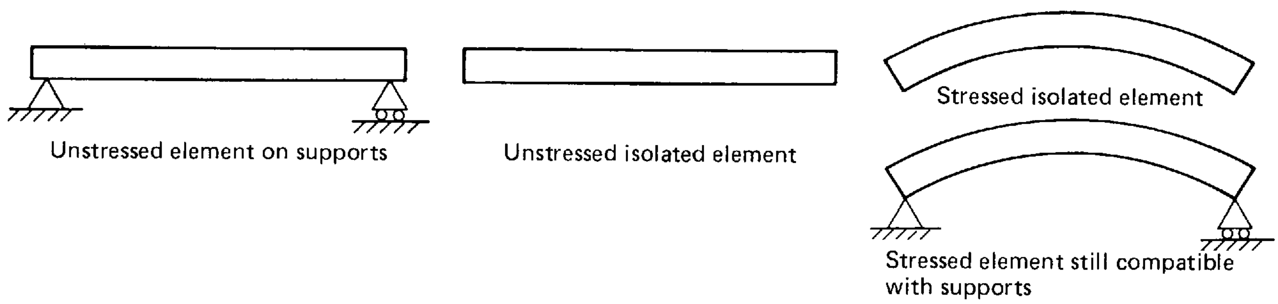


Figure 32: Prestressed element as part of a statically determinate structure

But when the element forms part of an indeterminate structure, the changes in shape resulting from prestressing will modify the support reactions. Additional reactions are required to make the prestressed member pass through support points and have suitable orientation where appropriate (Figure 33).

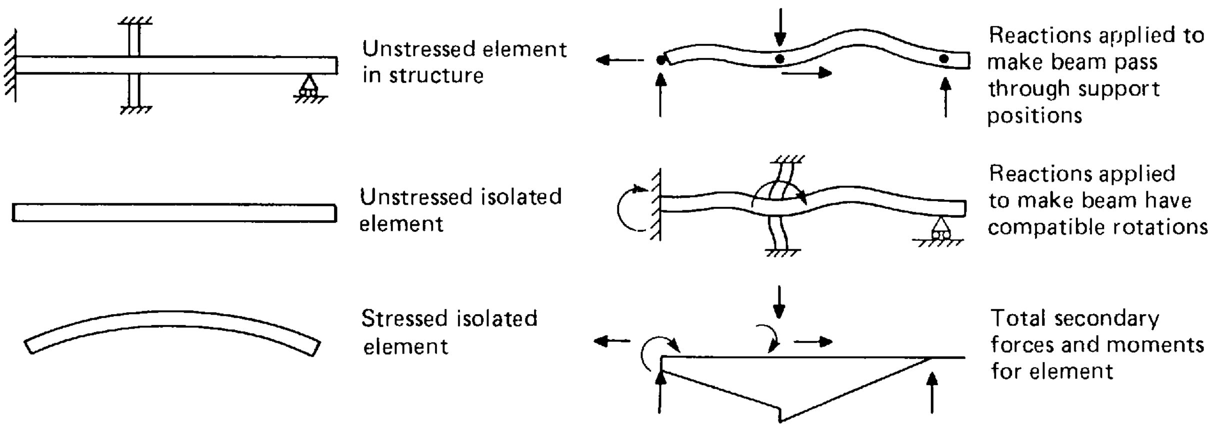


Figure 33: Reactions on a prestressed element due to secondary effects

These secondary reactions result in secondary forces and moments in the members. These are typically constant axial and shear forces throughout a span and uniformly varying moments. The calculation of these secondary effects can be difficult when staged construction, creep and shrinkage are considered. (Note that secondary effects cannot develop in cantilevers as they are statically determinate.) Methods of calculating secondary effects are given in Appendix D.

Equivalent loads will automatically generate the primary and secondary effects when applied to the structure.

Serviceability calculations do not require any separation of the primary and secondary effects, and analysis using the equivalent loads is straightforward. However, at Ultimate Limit State the two effects must be separated because the secondary effects are treated as applied loads. The primary prestressing effects are taken into account by including the tendon force in the calculation of the ultimate section capacity. The primary prestressing forces and moments must therefore be subtracted from the equivalent load analysis to give the secondary effects.

To calculate the ultimate loading on an element, the secondary forces and moments are combined with the ultimate forces and moments from dead and live loads. The Handbook to BS8110[17], suggests that the partial load factor on secondary effects should be 1.0. The total ultimate moments can be redistributed in accordance with BS8110, Part 1, Section 4.2.3[4].

6.10 *Flexural section design*

6.10.1 *Serviceability Limit State after all losses*

The bending moments calculated from the critical loading conditions given in Section 6.5, including the tendon effects, provide the serviceability stresses at each section using:

$$\text{top fibre stress, } f_t = \frac{P}{A_c} + \frac{M}{z_t}$$

$$\text{bottom fibre stress, } f_b = \frac{P}{A_c} - \frac{M}{z_b}$$

where: z_t = the top section modulus
z_b = the bottom section modulus
M = the total out-of-balance moment

$$M = M_A - Pe + M_S$$

e = eccentricity of tendons, taken as positive below the neutral axis
M_A = applied moment due to dead and live loads
M_S = moment from prestress secondary effects

One-way spanning floors

Bonded tendons: The maximum allowable concrete compressive and tensile stresses for floors with bonded tendons are given in BS8110, Part 1, Section 4.3.4.2 and 4.3.4.3[4] respectively. Most buildings will be satisfactory as Class 3 structures and the nature of the loading must be considered when deciding on a 0.1 or 0.2mm crack width (e.g. frequency and duration).

Unbonded tendons: The maximum allowable concrete compressive stresses in floors with unbonded tendons are as for floors with bonded tendons and are given in BS8110, Part 1, Section 4.3.4.2[4]. The maximum concrete tensile stresses should be taken as those given for group (b) in Table 4.2 of the Standard, with a limiting crack width of 0.1mm. These values must be adjusted for section depth as given by Table 4.3 of the Standard. If the stresses are enhanced by increasing the un-tensioned reinforcement as is allowed for bonded tendons in BS8110[4], crack widths and deflections should be rigorously checked. All concrete tension shall be carried by un-tensioned reinforcement (see Section 6.10.5).

Flat slabs (two-way spanning)

Flat slabs may be analysed in either of two ways. The more common method is to analyse equivalent frames in each direction. In this case some account must be taken of the peaking of the moments at the columns, described in Section 2.4. The analysis results in moments and stresses averaged across the width of the panel. The stresses should be limited to those given in Table 2.

Grillage or finite element analysis may be used, but this is normally only justified with floors of unusual configuration or where a design is to be constructed many times, such as in a high-rise building. If such analytical techniques are used which take into account the distribution of moments and stresses across a panel, then the allowable stresses given for one-way spanning floors may be used. Particular care must be taken in modelling the column/floor intersection and in the interpolation of the results obtained.

Location	In Compression	In Tension	
		with bonded reinforcement	without bonded reinforcement
Support	$0.24\sqrt{f_{cu}}$	$0.45\sqrt{f_{cu}}$	0
Span	$0.33\sqrt{f_{cu}}$	$0.45\sqrt{f_{cu}}$	$0.15\sqrt{f_{cu}}$

Table 2: Allowable average stresses in flat slabs, (two-way spanning), analysed using the equivalent frame method.

Note: Bonded reinforcement may be either bonded tendons or un-tensioned reinforcement.

In Table 2, the support zone shall be considered as any part of the span under consideration within $0.2 \times L$ of the support, where L is the effective span. Outside of this zone is considered to be the span zone.

Additional designed un-tensioned reinforcement is required in the support zone of all flat slabs, and in the span zone of slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$. The design of this reinforcement is presented in Section 6.10.5.

6.10.2 Transfer condition

Transfer stresses should be checked for all floors. These are likely to be more onerous for floors with high imposed loads.

Un-tensioned reinforcement shall be calculated in a similar manner to the reinforcement for the Serviceability Limit State (see Section 6.10.5).

One-way spanning floors

BS8110, Part 1, Clause 4.3.5.1[4] gives suitable limits for one-way floor concrete compressive stresses at transfer of $0.5f_{ci}$ at the extreme fibre (or $0.4f_{ci}$ for near uniform stress distribution) where f_{ci} is the concrete strength at transfer. Clause 4.3.5.2 gives the limits for allowable concrete tensile stresses which, for most buildings, will be $0.36\sqrt{f_{ci}}$.

Flat slabs (two-way spanning)

The allowable stresses given in Table 2 for the Serviceability Limit State also apply to the transfer condition for slabs analysed using the equivalent frame method, however, f_{cu} should be substituted by f_{ci}. For slabs analysed by the grillage or finite element methods, the allowable stresses are those given for one-way spanning floors.

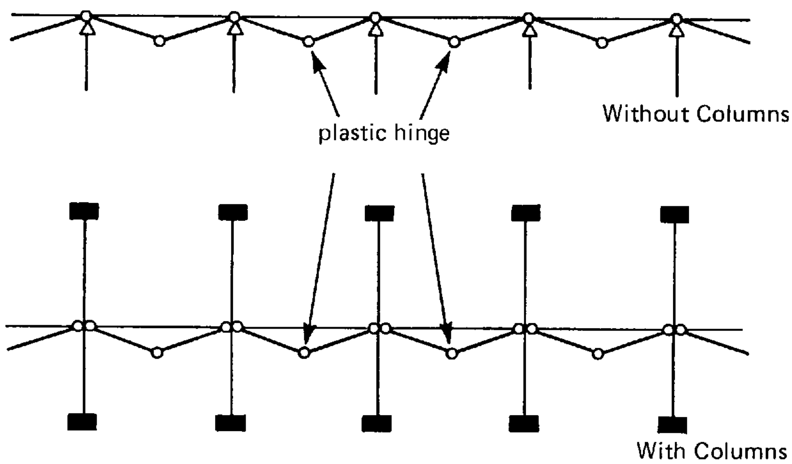
6.10.3 Ultimate Limit State

An Ultimate Limit State check is necessary on all floors in addition to the Serviceability Limit State previously covered. In this condition, the factored dead and applied loads are considered together with the secondary effects of the prestressing (see Section 6.9). The primary prestress effects are considered as part of the section strength. Additional un-tensioned reinforcement may be required in order to generate an adequate moment capacity.

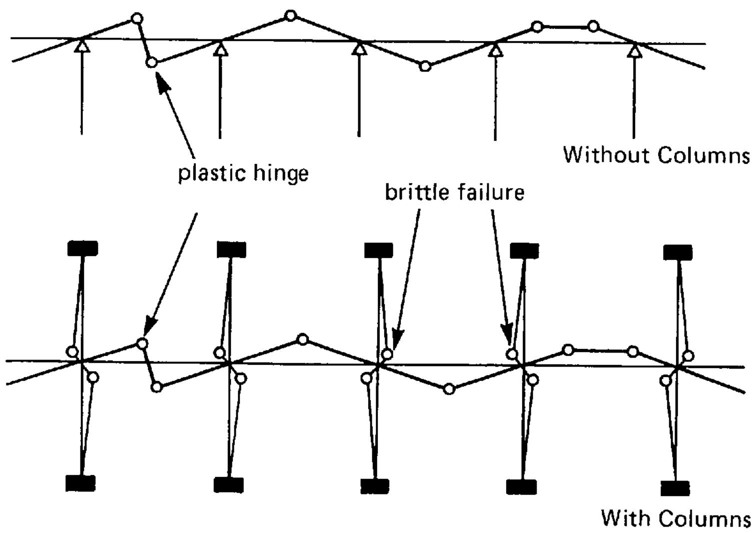
BS 8110, Part 1, Section 4.3.7[4] gives guidance on the assumptions for calculating the concrete and un-tensioned reinforcement stresses and the allowable design stresses for the tendons. In the above Section, equation 52 for unbonded tendons has been developed from the results of tests in which the stress in the tendons and the length of the zone of inelasticity in the concrete were both determined. The floor is considered to develop both elastic and inelastic zones and the length of the inelastic zone is taken to be 10 × the neutral axis depth.

The extension of the concrete at the level of the tendons is assumed to be negligible in the elastic zones and the extension in the inelastic zone is assumed to be taken up uniformly over the length, l, of the tendon. This is discussed further in references 29 and 30.

Hence, for a simply supported floor there is only one inelastic zone associated with the failure, but with a continuous floor the number of inelastic zones required for failure is more complex (see Figure 34). The length of tendon, l, in equation 52 can be modified, bearing in mind that if the tendon does not continue the full length of the continuous floor it may not include all the inelastic zones necessary for failure. It is therefore prudent to assume no more than one inelastic zone per span, and no more than two inelastic zones for the full length.



(a) Ductile failure



(b) Brittle failure

Figure 34: Zones of inelasticity required for failure of a continuous member.

6.10.4 *Progressive collapse*

Where progressive collapse involves the use of unbonded tendons in key elements, the maximum stress in the unbonded tendon shall not exceed $0.85f_{pu}$. This ensures that the anchorages are not over-stressed, and protects against catenary action.

In unbonded members there is also the risk that if tendons are severed accidentally there will be a 'progression' of failure for the full length of the tendons. This is particularly relevant for one-way spanning members such as beams, ribs and slabs spanning onto beams or walls.

In the case of one-way members where horizontal progressive collapse is of concern, it is necessary to reinforce with un-tensioned steel. This should be provided to satisfy the load case of dead load plus one third live load [DL + (1/3)LL] with an overall load factor of 1.05, and reduced material factor in accordance with BS8110, Part 1, Clause 2.4.3.2[4] for 'effects of exceptional loads or localised damage'. Reinforcement should be in accordance with normal BS8110 limits and arrangements.

Experimental and practical evidence in the USA has established that this problem does not occur in the internal bays of flat slabs due to the overall 'plate' or membrane action. The possibility of horizontal progressive collapse of edge and corner panels of flat slabs must be considered. These panels should be supported for the situation where the tendons parallel to the edge have been severed. This support can typically be provided by bonded reinforcement in the panel or an edge beam.

6.10.5 *Designed flexural un-tensioned reinforcement*

Additional un-tensioned reinforcement shall be designed to cater for the full tension force generated by the assumed flexural tensile stresses in the concrete for the following situations:

- All locations in one-way spanning floors using unbonded tendons.
- All locations in one-way spanning floors where transfer stresses exceed $0.36\sqrt{f_{ci}}$.
- Support zones in all flat slabs.
- Span zones in flat slabs using unbonded tendons where the tensile stress exceeds $0.15\sqrt{f_{cu}}$.

The reinforcement shall be designed, with reference to Figure 35, to act at a stress of $(5/8)f_y$ as follows:

$$h-x = \frac{-f_{ct} \times h}{f_{cc} - f_{ct}}$$

The value of f_{ct} will be negative in tension

$$F_1 = \frac{-f_{ct} \times (h-x) \times b}{2}$$

$$A_s = \frac{F_t}{(5/8)f_y}$$

The reinforcement shall be designed for the stresses at Serviceability Limit State, both after all prestress losses and at transfer conditions. It shall be placed in the tensile zone, as near as practicable to the outer fibre (see Section 7.5). Under transfer conditions any designed reinforcement is likely to be on the opposite face to that required after all losses.

At Ultimate Limit State, additional un-tensioned reinforcement may also be required (see Section 6.10.3). Any reinforcement provided for the Serviceability Limit State may also be used in the calculation of the moment capacity at Ultimate Limit State.

The designed reinforcement shall be checked against the minimum requirements given in Section 6.10.6.

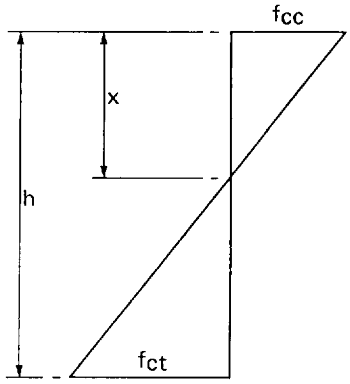


Figure 35: Section stresses used for the calculation of un-tensioned reinforcement.

6.10.6 Minimum un-tensioned reinforcement

Where fire ratings of greater than 2 hours are required, it is recommended that anti-spalling reinforcement be placed in the soffit when no other reinforcement is provided.

One-way spanning floors

Bonded tendons: There are no minimum un-tensioned reinforcement requirements for one-way spanning floors with bonded tendons. It is considered that these floors have sufficient tendon-to-concrete bond to distribute flexural cracking. Care should be taken to ensure sufficient reinforcement is provided to guard against cracking before stressing, if early phased stressing is not employed.

Unbonded tendons: One-way spanning floors with unbonded tendons should have minimum reinforcement in accordance with BS8110, Part 1, Table 3.27, Figures 3.24 and 3.25[4]. This reinforcement should be spread evenly across the full width of slab in accordance with the spacing rules given in BS8110, Part 1, Section 3.12.11[4].

Flat slabs (two-way spanning)

All flat slabs shall have minimum un-tensioned reinforcement at column positions to distribute cracking. The cross-sectional area of such reinforcement shall be at least 0.075% of the gross concrete cross-section ($0.00075 \times A_c$), and shall be concentrated between lines that are 1.5 times the slab depth either side of the width of the column. The reinforcement shall be placed as near as practical to the top of the floor, with due regard for cover and tendon location, and shall extend at least $0.2 \times L$ into the span or as far as necessary by calculation (see Section 6.10.1 and 6.10.2). The maximum pitch of the reinforcement should be 300mm.

In the span zone, there are no minimum requirements. However, when unbonded tendons are used it would normally be necessary to provide designed un-tensioned reinforcement in the bottom of the slab (see Section 6.10.1). This reinforcement should extend at least to within a distance of $0.2 \times L$, measured from the centre of the support. It should be placed at a spacing of $3 \times$ slab thickness or 500mm, whichever is the lesser.

Slab edges

Un-tensioned reinforcement should be placed along edges of all slabs. This should include U-bars laced with at least two longitudinal bars top and bottom, as shown in Figure 38. See also Section 6.12. Reinforcement should be provided in the triangular unstressed area between anchorages. See Section 6.13.

6.11 Shear strength

6.11.1 Beams and one-way spanning slabs

The method in BS8110, Part 1, Section 4[4] should be used.

Where unbonded tendons are used, the value of v_c in equation 55 of BS8110[4] should be reduced by a factor of 0.9 as recommended by Regan[19].

6.11.2 Flat slabs (punching shear)

BS8110[4] does not provide specific guidance for checking punching shear for prestressed flat slabs. The working party considered a number of different methods while preparing this handbook, with a view to satisfying the following aims:

- Design capacities to be in line with other international standards.
- Increased punching shear capacity for bonded tendons.
- Increased punching shear capacity when tendons are concentrated in the vicinity of the column.
- A design method which complements BS 8110[4] as far as possible.
- A design method which allows a smooth transition from reinforced concrete to prestressed concrete and allows for situations where the slab is prestressed in one direction only.

The following method achieves these aims and is recommended.

Calculate the effective shear force, V_{eff}, in accordance with BS8110, Clause 3.7.6.

The shear resistance, V_c is obtained by adding together the contributions from each of the sides of the critical shear perimeter as given in BS8110, Clause 3.7.7[4]. The shear resistance of each side of the critical perimeter should be calculated in accordance with BS8110, Clauses 4.3.8 and 4.4[4] as modified below.

Flat slabs are generally not heavily prestressed and will therefore be governed by the design for "sections cracked in flexure", using equation 55 (BS8110, clause 4.3.8.5[4]). Equation 55 does not, however, provide a smooth transition from reinforced to prestressed concrete because of the term:

$$\left(1 - 0.55 \frac{P_{pe}}{P_{pu}}\right)$$

For lightly prestressed structures the inclusion of this term in equation 55 can lead to a shear capacity less than that which would be calculated for the same slab but without prestress. This is obviously incorrect. The British Cement Association[31] has recently compared various forms of shear calculation with published test results and concluded that equation 55 would be more consistent with the test results if the above term were omitted. It is therefore recommended that the shear resistance of each side of the critical perimeter be calculated from equation 55 modified as follows:

$$V_{cr} = v_c b_v d + M_o \frac{V}{M}$$

where v_c, b_v and d are the values for the relevant side of the critical perimeter.

The value of v_c should be calculated taking into account both A_s and A_{ps} for bonded tendons in accordance with BS8110 Clause 4.3.8.1. However the presence of unbonded tendons should be neglected in this calculation of v_c. No further reduction is considered necessary (e.g. as suggested in reference 17, page 98).

The de-compression moment, M_o, should be calculated for the width of the side of the critical perimeter under consideration. It should be noted that the axial effects of prestress, P/A_c, are uniformly distributed over the width of the slab whereas the prestress moment effects ($P_e + M_s$) are concentrated at the location of the tendons at the critical perimeter. Hence the two contributions to M_o have to be calculated separately as follows (for a hogging moment region):

$$M_o = 0.8PZ_t^*/A_c - 0.8P^*e^*$$

where:

0.8	=	a safety factor on prestress (BS8110, Clause 4.3.8[4])
P	=	the total prestress force, over the full panel width, after all losses
A_c	=	the concrete section area across the full panel width
Z_t^*	=	section modulus for the top fibre over the width of the side of the critical perimeter
P^*	=	the total prestress force for all tendons passing through the side of the critical perimeter
e^*	=	the eccentricity of the prestress force, P^*, at the critical perimeter, measured positive below the centroid

The value of V/M must be calculated for the load case under consideration, normally that which generates the largest V_{eff}. V/M should strictly be calculated at the location of the critical perimeter but may be calculated conservatively at the column centreline. For a typical internal column V/M will vary from 5.5/L to 6.0/L, depending on the ratio of dead load to live load, where L is the span length.

6.11.3 Openings in slabs

Tendons should be continuous and displaced horizontally to avoid small openings. If tendons are terminated at the edges of large openings, such as at stairwells, an analysis should be made to ensure sufficient strength and proper behaviour. Edges around openings may be reinforced similarly to conventionally reinforced slabs; in the case of large openings, supplementary post-tensioning tendons may be used to strengthen the edges around openings.

6.12 Anchorage bursting reinforcement

Reinforcement is usually required to resist the tensile stresses caused by the concentration of the forces applied at the anchors. At some distance from the edge of the floor (or the anchorages) it can be assumed that the distribution of stresses is the classic linear distribution and depends only on the magnitude and position of the resultant of the forces applied to the edge of the floor.

Between the edge and the above plane the lines of force are curved and give rise to transverse tensile stresses in both directions perpendicular to the applied force direction.

Figures 36 and 37, adapted from reference 18, illustrate the varying proportions of the prestressing force manifesting itself as a splitting tensile force of magnitude depending on the anchorage and floor relative geometries.

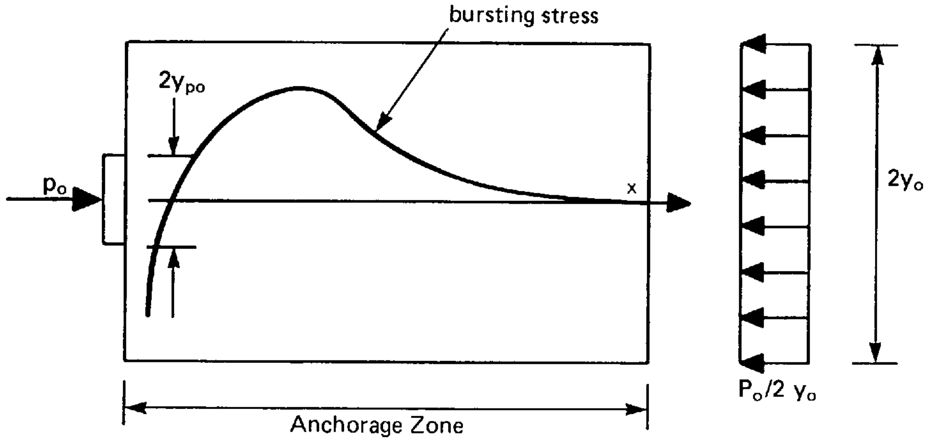


Figure 36: Bursting stresses in rectangular beam subjected to an axial symmetric force.

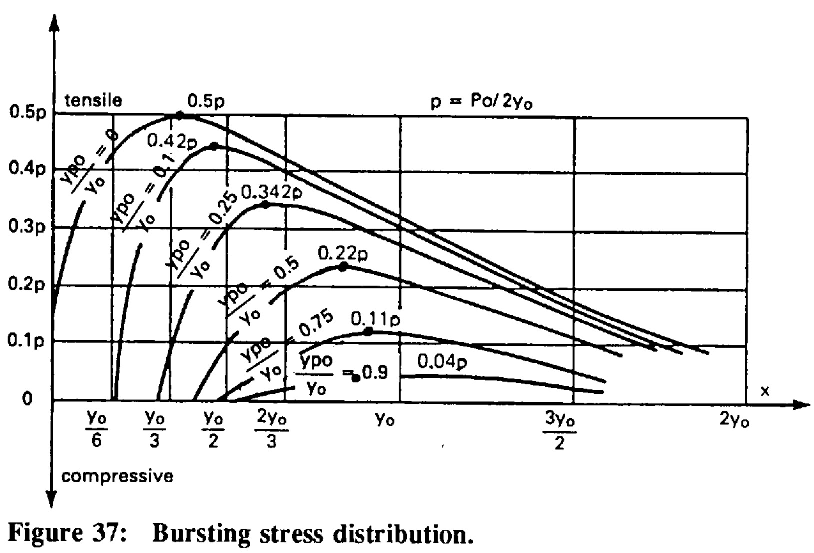


Figure 37: Bursting stress distribution.

Where a group of anchorages exist, as is often the case for 'banded' slab tendons, the bursting stress zones for both the individual and collective anchorages should be considered, and reinforcement placed accordingly.

Care should also be taken to ensure that the phasing of the application of prestress to anchorage groups does not create a bursting condition which may be critical. If this condition is unavoidable, reinforcement should be added accordingly.

BS8110, Part 1, Section 4.11[4] gives design bursting tensile forces of a similar nature to Figure 35 and limits the steel stress to $200N/mm^2$ at Serviceability Limit State. It is suggested that bars with $f_y = 460N/mm^2$ are used for this reinforcement. Alternatively the bursting forces and distribution may be calculated in a more rigorous method, such as suggested by Guyon[20]. In some cases it may be shown that the concrete is capable of withstanding bursting without the addition of reinforcement. At Ultimate Limit State for unbonded tendons only, reinforcement requirements should be checked in accordance with BS8110, Clause 4.11.3. This Ultimate Limit State check is unlikely to be governing.

Where anchorages are grouped, or where the distribution of anchorages does not reflect the distribution of concrete in the cross-section, it may be necessary to include 'equilibrium' reinforcement to prevent splitting between anchorages. Also when anchorages occur within the plan area of the floor rather than at the perimeter, it may be necessary to include 'following' reinforcement. This reinforcement runs parallel to the tendon past the anchorage to limit cracking adjacent to the anchorage. These effects are discussed in CIRIA Guide No. 1[21].

Post-tensioning system suppliers often test their anchorage systems in concrete prisms, reinforced in a similar manner to that encountered in practice and using a prism size similar to the common on-site member thickness, etc. Such tests may be deemed under BS8110, Part 1, Section 2.6[4] to satisfactorily model the on-site conditions and the reinforcement may be considered adequate provided suitable safety factors are observed[14].

Two examples showing the calculation of, and the detailing of, bursting reinforcement are given in Appendix E.

6.13 Reinforcement between tendon anchorages

Figure 43 shows an area of slab between tendon anchorages which require reinforcement to span the unstressed zones. Any prestressed tendons which pass through this zone, parallel to the slab edge, may be included with the relevant reinforcement, provided it is in the local tension zone.

The area of tension reinforcement (and/or prestressed tendons) provided parallel to the slab edge should resist bending moments from the ultimate vertical loads calculated for a continuous slab spanning l_a. This reinforcement should be evenly distributed across a width equal to $0.7l_a$, and should be continuous along the edge.

The area of reinforcement placed perpendicular to the slab edge should be the greater of 0.13%bh, or a quarter of the reinforcement provided parallel to the edge. It should be placed evenly between anchorages, and extend the greater of l_a or $0.7l_a$ plus a full anchorage length into the slab.

6.14 *Deflection and vibration*

Deflection

This is a Serviceability Limit State relating to the complete structure. The deflections of a structure, or of any parts of a structure, should not adversely affect appearance or performance.

The final calculated deflection (including the effects of temperature, creep and shrinkage, and camber), measured below the line between the supports of the floor and roof, should not in general exceed span/250.

In addition, where internal partitions, cladding and finishes can be affected by deflection, the deflections should be limited in accordance with BS8110, Clause 3.2.1.2[4].

As a guide for a prestressed solid slab, continuous over two or more spans in each direction, the span/depth ratio should not generally exceed 42 for floors and 48 for roofs. These limits may be increased to 48 and 52 respectively, if detailed calculations show acceptable behaviour with regard to short- and long-term deflections, camber and vibration. Lower span/depth ratios will often apply to slabs with high live/dead load ratios. The span/depth ratios for waffle slabs should not generally exceed 35.

Vibration

Prestressed floors are usually thinner or span further than unprestressed floors. They therefore tend to have lower natural frequencies and greater consideration must be given to their dynamic performance.

The Steel Construction Institute has published a design guide on the vibration of floors[22]. This guide covers sources of vibration excitation in buildings, human reaction to vibration, evaluation of natural frequencies, response of floors and design procedures.

Although it was written primarily for checking the acceptability of lightweight concrete composite floors on steel beams, most of the guide is relevant for any floor system. Appendix G gives a procedure, based on the guide for checking prestressed floors with a rectangular grid. Vibration should not be a problem for general office buildings if the total slab depth is greater or equal to the values given in Table 1.

For more sensitive locations, or for slabs shallower than the above criteria, an assessment of the dynamic response of the floors should be made.

6.15 *Lightweight aggregate concrete*

Additional considerations on the use of lightweight aggregate concrete are given in BS8110, Part 2, Section 5[4], and the Guide to the Structural Use of Lightweight Aggregate Concrete[23].

The allowable tensile stresses given in Sections 6.10.1 and 2 should be reduced by a factor of 0.8; however, the allowable compressive stresses need not be reduced.

In BS8110, Part 2, Clause 5.4[4], the maximum allowable shear stress is given as the lesser of $0.63\sqrt{f_{cu}}$ or $4N/mm^2$. These values are 0.8 times the values given for normal weight concrete. However, this limit is related to compressive strut failure, not to tensile failure. In the view of the Working Party the limitations for normal weight concrete, the lesser of $0.8\sqrt{f_{cu}}$ and $5N/mm^2$, may also be used for lightweight concrete.

7. DETAILING

(Reference should also be made to "Standard method of detailing structural concrete"[24])

7.1 Tendon distribution

Various methods for distributing the tendons can be used. These are discussed in Section 2.4.

In situations of overload, tendons passing through the column/floor intersection are more effective than tendons elsewhere. It is therefore recommended that a minimum of two tendons should pass through this section.

For ribbed slabs or beams, the distribution of tendons is dictated by the spacing of members.

7.2 Tendon spacing

The maximum spacing of uniformly distributed tendons should not exceed six times the slab depth for unbonded tendons or eight times the slab depth for bonded tendons.

Unbonded tendons may be placed in groups if required. It is recommended that grouped tendons are laid side by side and do not exceed four tendons per group.

The minimum horizontal distance between ducts or groups of tendons should be the greater of 75mm or the group/duct width.

Should it be necessary to arrange the tendons in vertical layers in beams or ribs, then it is recommended that the gap between the layers should be at least the vertical dimension of the tendon or duct. In the case of bonded tendons where oval metal ducts are used, it is recommended that their positions are staggered to ease the placing of concrete.

If tolerances on tendon positions are not stated, the values in Table 3 should be adopted.

Slab thickness	Tolerances	
	Vertically	Horizontally
$h < 200mm$	$\pm h/40$	$\pm$ 20mm
$h > 200mm$	$\pm$ 5mm	$\pm$ 20mm

Table 3: Tolerances on tendon positioning

7.3 *Tendon notation*

The accepted standard notation of tendons on drawings is shown in Figure 38. It is recommended that this legend Figure is included on all tendon layout drawings.

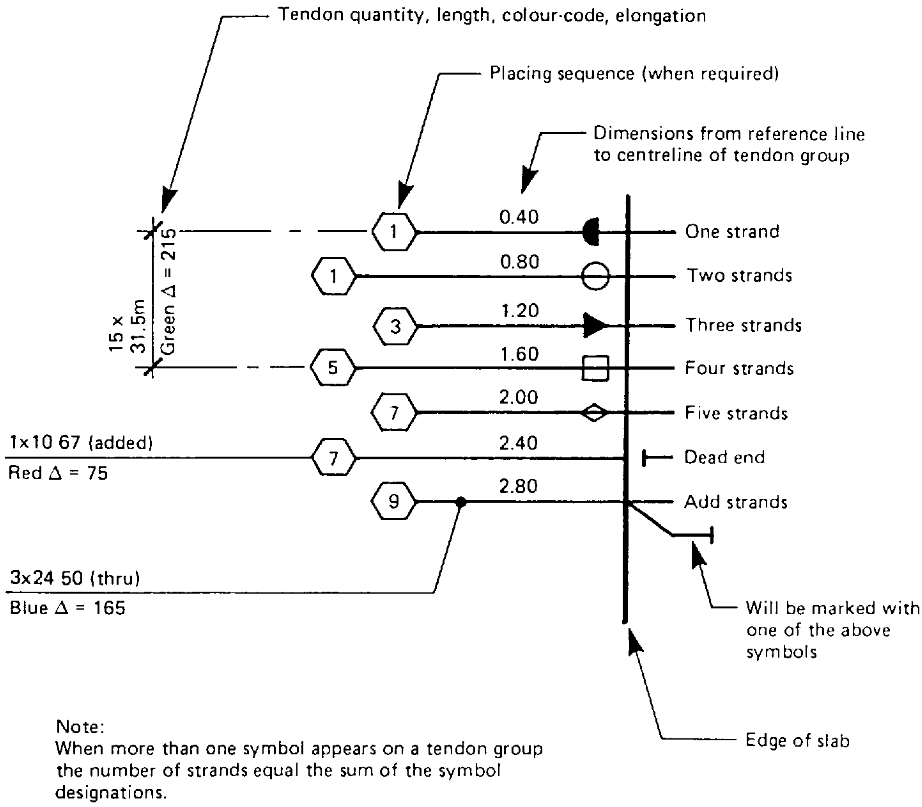


Figure 38: Method of notation for use on tendon layout drawings.

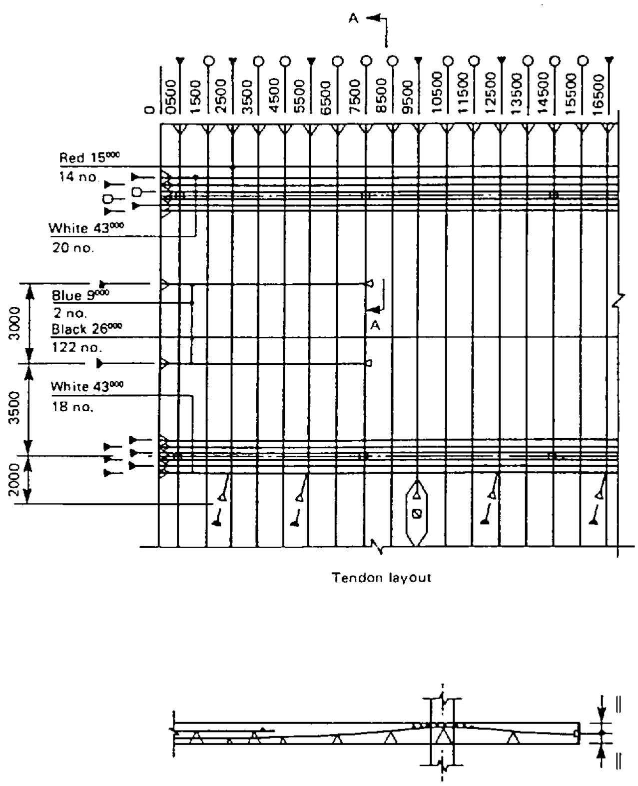
Figure 39a shows an example using the legend showing groups of tendons and anchorages types, together with the tendon sequence, detailed. This Figure is based upon reference 24 modified along lines recommended in this document.

Tendon profiles in the longitudinal and transverse directions are shown using an exaggerated scale for the vertical dimensions. These are usually given from the soffit of the slab to the centreline of the duct/sheath and are plotted at intervals of 1m. Closer centres may be necessary for sharp vertical curves. For ease of placement on site, shop drawings are detailed giving the vertical tendon position from soffit to underside of tendon.

7.4 *Tendon supports*

The profile of the tendons is critical to the floor performance. It is therefore recommended that the support centres do not exceed 1m. For ribbed slabs or beams, support bars can be adequately held by firm wire ties. Spot welding can be used but this makes any adjustment difficult.

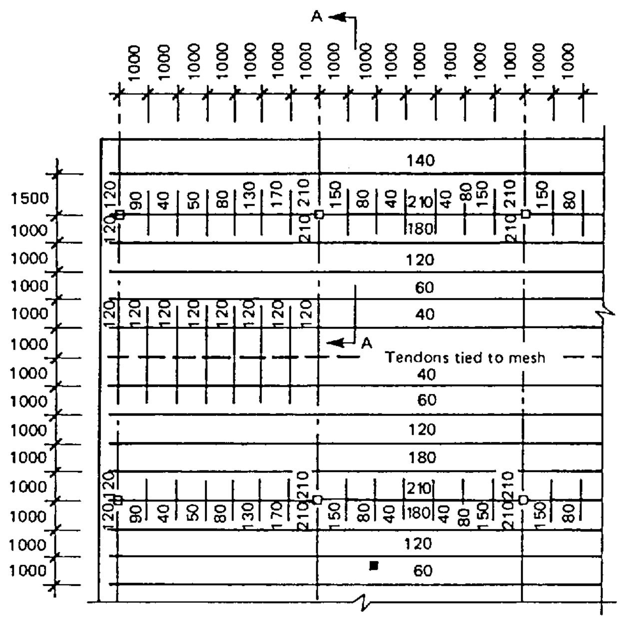
Figure 39b shows a typical support bar layout. The actual layout may be modified by the contractor depending on the support system adopted, so that the specified tendon profiles are attained and adequate support is provided.



Placing sequence not shown

Section A.A

(a) Flat slab tendon layout

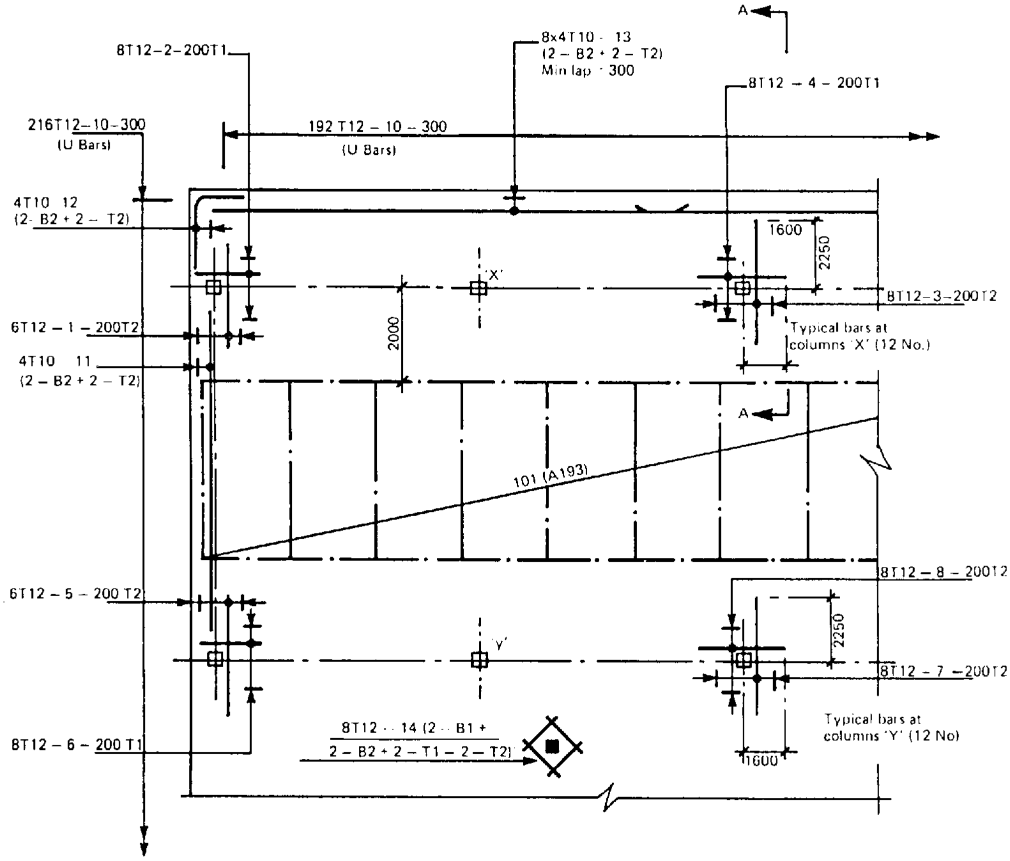


Note:

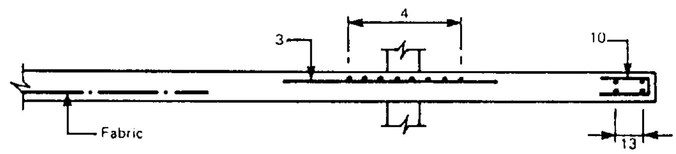
1. Height given is from soffit of slab to underside of tendon
2. Diameter of support bar is 10mm.

(b) Typical tendon profile and support bar layout for a flat slab

Figure 39: Flat slab tendon and support layout detailing.



Floor plan



Section A.A showing reinforcement details

Figure 40: Flat slab reinforcement layout.

7.5 *Layout of un-tensioned reinforcement*

Figure 40 shows an example of the reinforcement that is always required at edges and in the top of flat slabs at columns. It also shows the reinforcement needed in the bottom of the slab at midspan for some design applications. See Section 6.10 for details.

7.5.1 *At columns*

Reinforcement should be placed in the top of the slab over columns. The design of such reinforcement is described in Section 6.10.5 with minimum requirements given in Section 6.10.6. Figure 41 shows a typical arrangement of tendons and un-tensioned reinforcement around a column.

Figure 41: Reinforcement arrangement at a column.

7.5.2 *Shear reinforcement*

Shear reinforcement in flat slabs, if required, is usually in the form of links or hairpins, although prefabricated shear reinforcement is available. Fabricated steel shear heads may also be used. See Figures 41 and 42 and Section 6.11 for details.

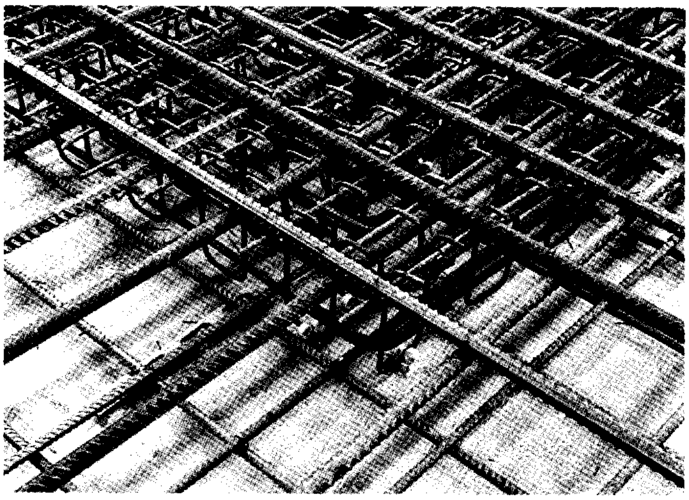


Figure 42: Prefabricated shear reinforcement.

7.5.3 *At and between anchorages*

An adequate amount of reinforcement should be placed at anchorage end blocks to avoid splitting of the concrete. A sample calculation to determine the amount of this reinforcement is given in Appendix F.

Reinforcement should be provided in the 45° wedge area between the anchorages (Figure 43).

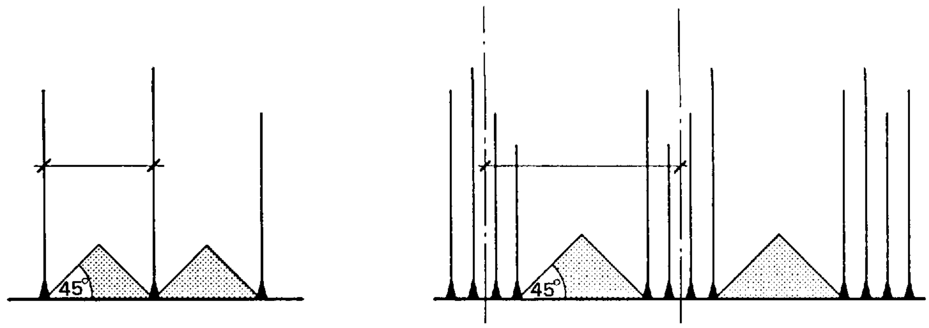


Figure 43: Unstressed areas between tendons requiring reinforcement.

7.6 Penetrations and openings in floors

Unbonded tendons may be diverted around the openings as they are relatively flexible (see Figure 44). The change of direction of the tendon should occur away from the opening, and trimmer bars should be provided to avoid possible cracking at the corners.

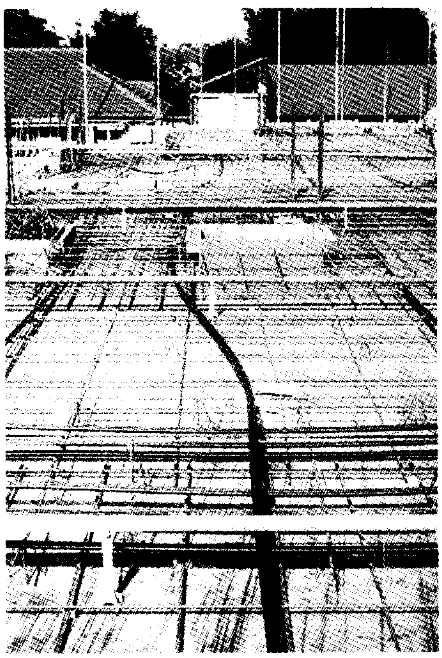


Figure 44: Unbonded tendons diverted around an opening.

The oval sheathing used in bonded tendons is very rigid in the transverse direction, and cannot be bent around openings. In this instance, openings should be confined to the areas between tendons.

The cutting of penetrations in finished slabs is not a problem in ribbed slabs where the tendon positions are, in effect, defined. Grouted tendons, providing the grout is effective, can be cut without significant loss of prestress. However, when unbonded tendons have been used, care must be taken to locate the tendons before concrete removal. Tendons can be cut and reinstated but it is recommended that this work be carried out by a specialist.

7.7 *Construction details*

7.7.1 *Extent of pours*

With bonded tendons, friction losses usually restrict the length of single end stressed tendons to 25m, and double end stressed to 50m. The lower friction values for unbonded tendons extend these values to 35m and 70m respectively. Longer lengths are achievable but the friction losses should be carefully considered.

These limitations usually determine the extent of pours. Prestressing tendons may be continuous through construction joints allowing larger areas without any permanent joints. Allowances should be made in accordance with good practice to accommodate temperature variations by the provision of expansion joints on larger slabs.

7.7.2 *Construction joints*

Generally construction joints should be made in the vicinity of quarter and third points of the span from supports.

Shear provision in accordance with good practice should be made by the introduction of expanded mesh, by roughening the previously poured surface or by the introduction of a shear key.

In long slabs, intermediate anchorages may be introduced which allow the stressing to be continuous through the construction joint (see Figure 45). Alternatively infill strips can be used, but it should be noted that these will not be prestressed. These strips are cast after the stressing of the adjacent sections is complete (see Figure 46). This operation should be delayed for as long a period as is reasonable to reduce the effects of creep and shrinkage.

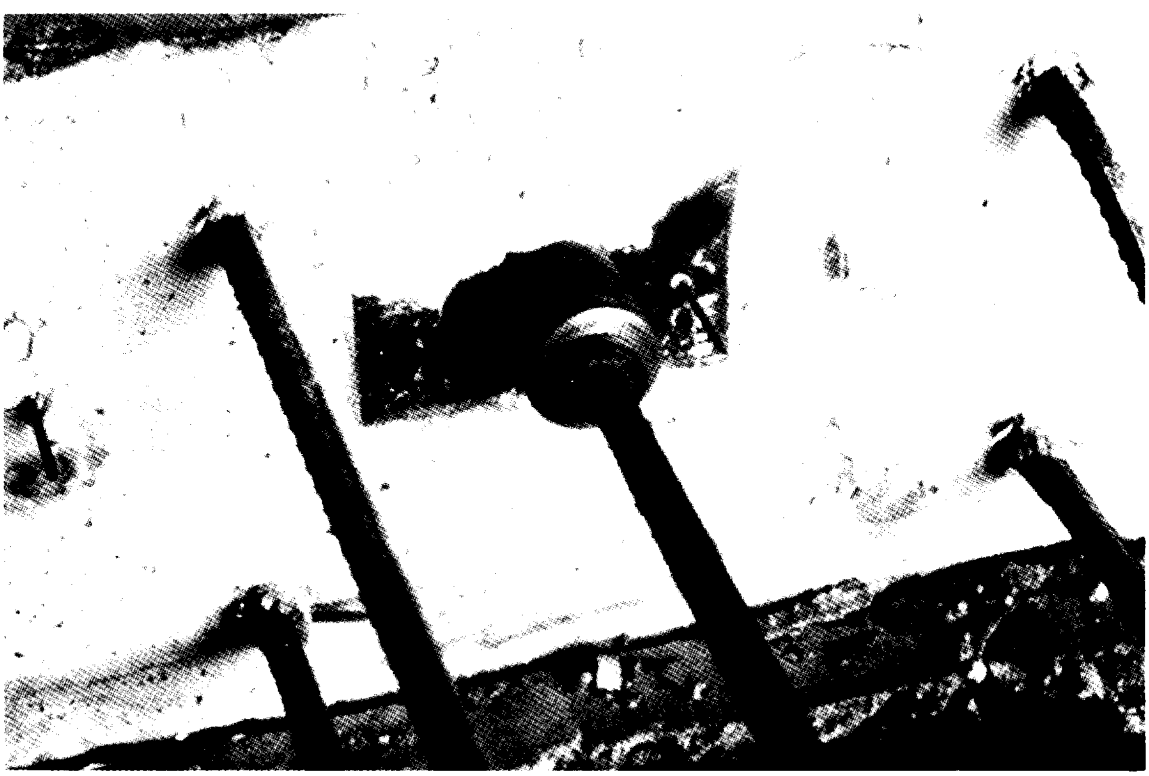


Figure 45: Intermediate anchor at a construction joint.

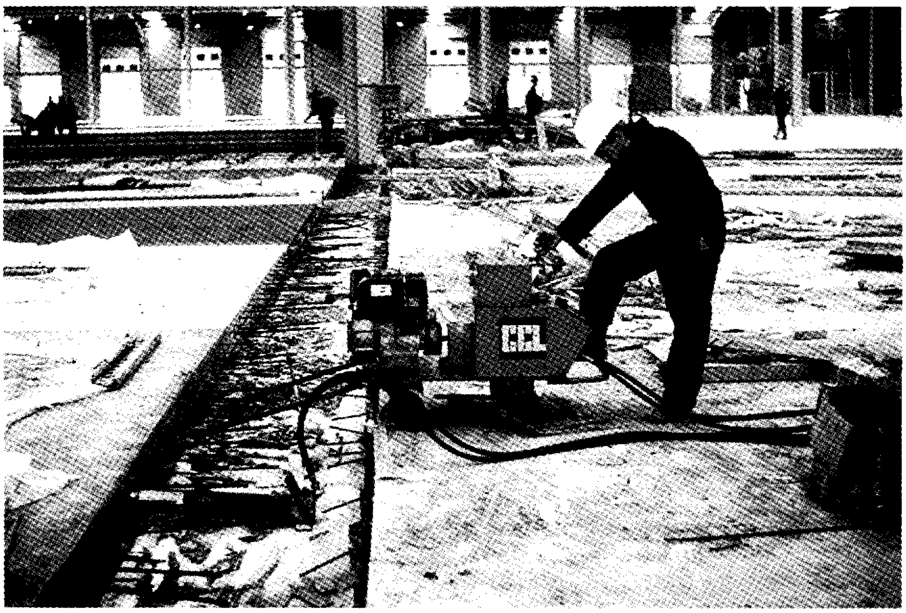


Figure 46: Infill strip for jack access.

In assessing the movement of slabs at expansion or contraction joints from the time of pouring concrete, a strain of 650×10^{-6} should be considered as normal. The drying out effect of air conditioning can increase this to 1000×10^{-6}.

The use of dowels perpendicular to the direction of stressing should be avoided as this could prevent the stress being transferred to the slab. Generally dowels should be avoided in slabs stressed in two directions.

7.7.3 *Protection of anchorages*

Tendons are normally anchored within the middle third of the slab to ensure adequate edge cover to the anchorage. Pocket formers at anchorages should be large enough to allow adequate trimming of the tendons after stressing, thus ensuring good end cover to the strand. Trimming should be carried out using a disc cutter or hydraulic shears (see Figures 47 and 48). In no circumstances should the tendon be trimmed by flame cutting. Pocket formers are normally proprietary plastic or polystyrene units which make up part of the anchorage fixings. Anchorages fixed to formwork are shown in Figure 49. It is recommended for unbonded tendons that, after trimming the strands, the wedges and the strand end are coated with grease of similar specification to that used in the tendon and that a watertight cap be applied over the coated area (see Figure 50). The minimum end cover to this cap should be 25 mm.

Figure 47: Strand trimming using a disc cutter.

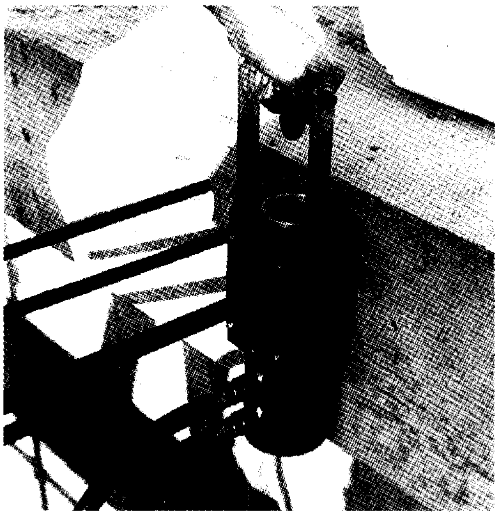


Figure 48: Strand trimming using purpose-made hydraulic shears.

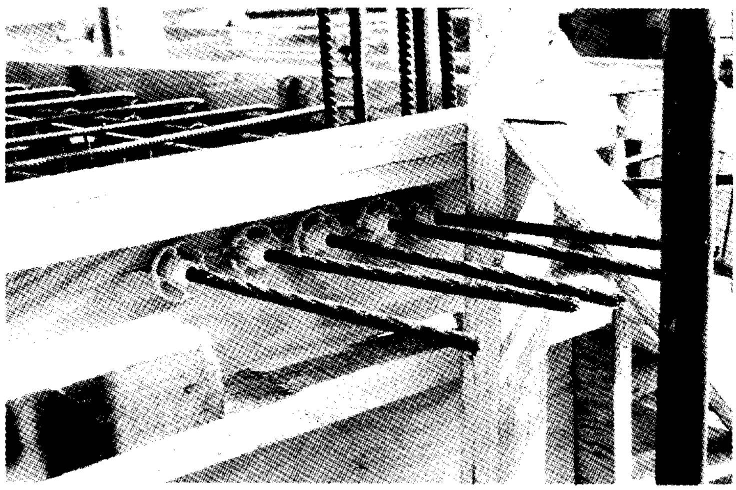


Figure 49: Anchorages for unbonded tendons: fixed to formwork.

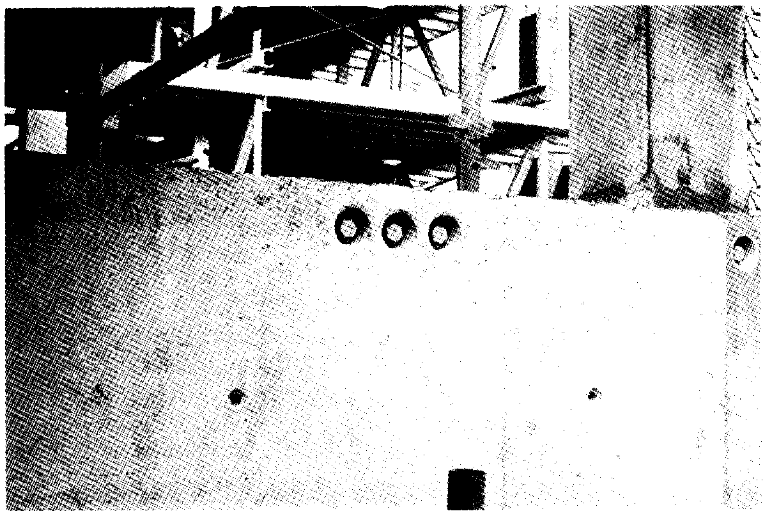


Figure 50: Grease-filled plastic cap to protect strand and wedge grips.

The pockets for anchorage are particularly vulnerable to ingress of moisture and it is therefore essential that they be properly filled with a non-shrink mortar as soon as possible after stressing is complete (see Figure 51). Before installing the pocket mortar, the concrete surfaces should be coated with a suitable bonding agent. In no circumstances should the mortar contain chlorides or other materials which could be harmful to the prestressing steel.

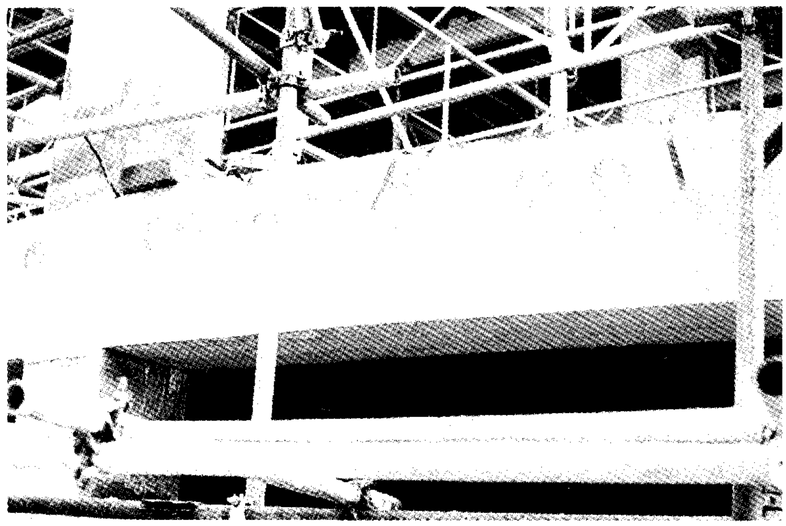


Figure 51: Anchorage block sealed with mortar.

7.7.4 *Back-propping*

Back-propping may be required to ensure that the construction loads can be safely carried by the earlier construction stages, and this must be considered by the designer in a similar manner to normal reinforced concrete construction.

7.7.5 *Stressing procedure*

The stressing forces, extensions and sequence should be specified on the drawings. This has to be planned in such a way that the prestress is applied as uniformly as possible, and that no overloading of the formwork occurs. The banded tendons are usually stressed first to ensure this is the case (see Figure 52). Wherever possible the use of different forces for tendons of the same size should be avoided.

Figure 52: Stressing banded tendons at slab edges.

In members where early stressing is desired to reduce the risk of early shrinkage cracking, it is common to stress the tendons in two stages. The first stage is usually 50% of the final prestress force, and is carried out as soon as the concrete has obtained adequate strength for the anchorage being used. This concrete strength is usually between 12 and 15 N/mm^2.

7.7.6 *Soffit marking*

Tendon positions in flat slabs are not apparent on completion of concreting. Recent practice has been to introduce soffit marking, where the cover to the tendon is less than the penetration of ceiling and service fittings. An illustration of typical marking is shown in Figure 53. Unpainted zones indicate no tendons. Dark zones indicate tendons near the soffit and white zones indicate tendons near the top of the slab.

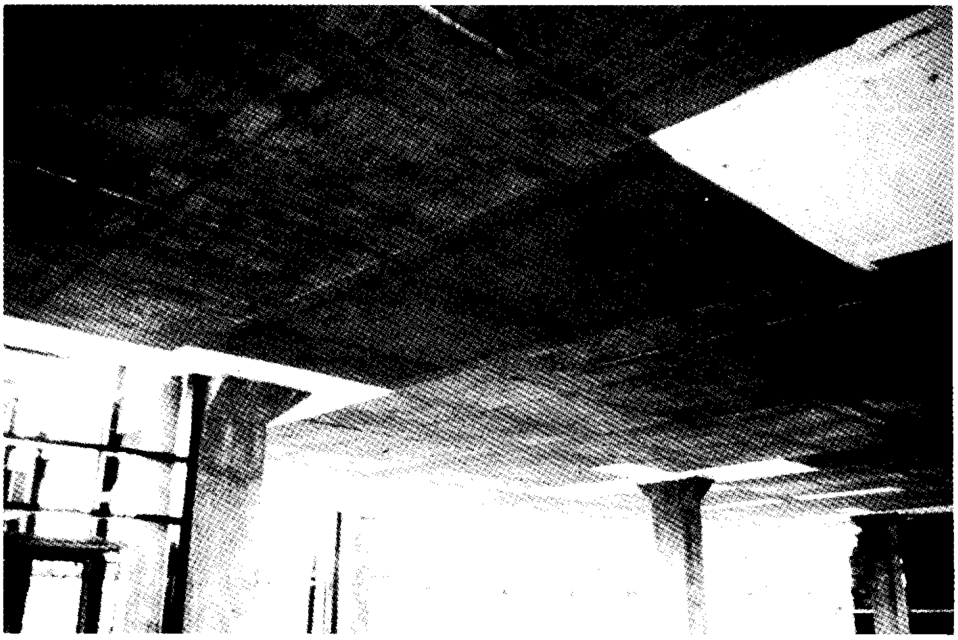


Figure 53: Soffit marking used to indicate tendon position.

8. DEMOLITION

8.1 General

Special precautions are required for the demolition of prestressed concrete structures, and it is recommended that the advice of a prestressing specialist is obtained before planning the demolition.

Two references giving useful information are the FIP guide to good practice[25] and the PTI publication on the recent demolition of a post-tensioned slab[26].

8.1.1 Structures with bonded tendons

These can normally be demolished using recognised methods of demolition. However, it is of fundamental importance that, during the initial stages, it is ascertained that the grouting is effective.

8.1.2 Structures with unbonded tendons

The energy introduced into the tendons in this instance is only secured by the anchorages. Release of this energy will occur over the complete length of the tendon no matter where it is cut. The sequence of releasing the tendons must be planned in detail to take into account the structure's ability to carry dead loads without prestressing and the introduction of temporary supports where necessary. Safety precautions should be taken near the anchorages although recent experimental work has shown that most of the energy is dissipated by friction, dislodging the wedges and breaking the concrete cover[27].

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APPENDICES

POST-TENSIONED CONCRETE FLOORS - DESIGN HANDBOOK

NOTE:

In some of the calculations given, there are instances where "rounded" figures have been shown in the equation but the result has been based on the "unrounded" figure.

APPENDIX A: Design Examples

INTRODUCTION

Two examples are given. The first is based on the use of unbonded tendons in a flat slab. The second on both bonded and unbonded tendons in a slab and beam arrangement spanning one way. The examples show differences which arise due to variations in structural form and tendon type, and illustrate the different design methods which can be adopted. The procedures used in the examples are widely adopted in the design of such structures today.

Contents of Appendix A

A1 *Example 1: Solid flat slab with unbonded tendons*

Equivalent frames are used to model the slab in each direction. Maximum design moments are obtained by a combination of live and dead load, with the equivalent prestress load from the tendons. Where tendon anchorages are away from the neutral axis, or are inclined to the neutral axis of the slab (either at the ends of the slab or within the span where tendons are 'stopped off'), their effect can be included in this method of analysis by the introduction of moments or point loads.

A floor plan of the building is shown in Figure A1 together with a typical transverse subframe. This example analyses subframes on gridlines 5 and B. Calculations are carried out for full bay width.

The structure is checked both at Serviceability and Ultimate Limit States. These checks are carried out at transfer, during construction (where typically, when two weeks old, the slab may be required to carry its own weight, plus the weight of the floor above at concreting, plus associated construction loads), and under working load conditions.

Properties

concrete:

f_{cu}	$= 40$ N/mm^2	
f_{ci}	$= 25$ N/mm^2	(strength at transfer)
E_c	$= 28$ kN/mm^2	(elastic modulus at 28 days)
E_{ci}	$= 21.7$ kN/mm^2	(elastic modulus at transfer from BS8110, Part 2, Section 7[(4)])

bonded reinforcement:

$f_y = 460$ N/mm^2

prestressing steel:

12.9 mm diameter superstrand with high-density polythene or polypropylene sheath and with lubrication/corrosion protection as detailed in Section 4.2.2.

P_k	$= 186$ kN	(characteristic strength of tendon)
A_{ps}	$= 100$ mm^2	(area of tendon)
f_{pu}	$= P_k/A_{ps} = 1860$ N/mm^2	(characteristic strength of prestressing steel)
E_{ps}	$= 195$ kN/mm^2	(elastic modulus)

Loading

Imposed loading:

finishes:	partitions		1.0 kN/m^2
	screed		1.2 "
	services in floor zone		0.5 "
	ceiling		<u>0.5</u> "
			<u>3.2</u> kN/m^2
live load:	typical office building	=	<u>4.0</u> kN/m^2
Total imposed loading		=	<u>7.2</u> kN/m^2

From Figure 16, a slab depth of 210 mm would be adequate. However, to reduce shear reinforcement requirements (see Figure 17), a depth of 225 mm is chosen.

Self-weight (using density of 24 kN/m^3)	=	<u>5.4</u> kN/m^2
Total dead load	=	<u>8.6</u> kN/m^2
Total live load	=	<u>4.0</u> kN/m^2

Check at temporary construction stage

construction load - self-weight of slab under construction above	=	5.4 kN/m^2
additional construction load	=	<u>1.5</u> "
		<u>6.9</u> kN/m^2

share this load between two lower floors by propping

load per floor	=	3.45 kN/m^2
self-weight of floor under design	=	<u>5.40</u> "
Total construction load per floor		<u>8.85</u> kN/m^2
worst loading = dead load + live load situation	=	<u>12.6</u> kN/m^2

Balanced load

At this stage in the calculation, it is recommended that the amount of load to be balanced is considered. The designer's experience can simplify this operation. In this example a balanced load consisting of all the dead load is chosen.
(Balanced loads are discussed in more detail in Section 6.7 of this report.)

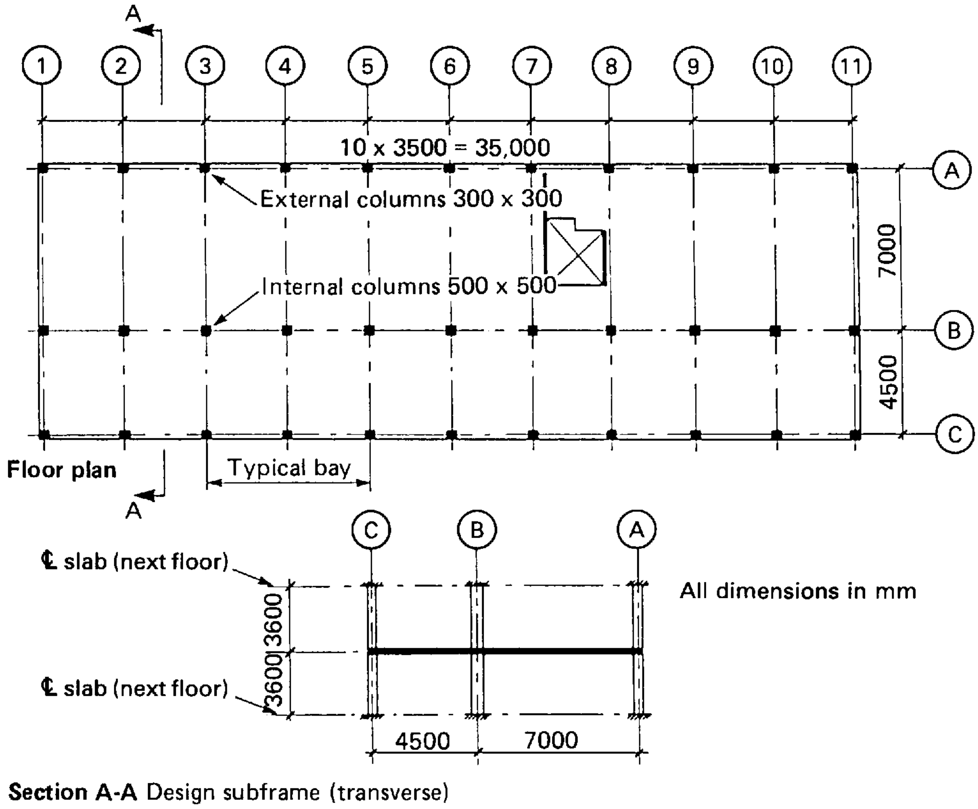


Figure A1: Floor plan and subframe for Example 1

Tendon profiles

Cover requirements in accordance with Section 5 of this report.

for adequate cover against corrosion	=	25mm
for 1.5 hours fire resistance	=	25mm

Take nominal cover to be <u>25mm</u>

The dimensions from the top surface of the slab to the tendons and reinforcing steel are shown in Figure A2. The positioning of the reinforcement must be considered at this stage, so as to obtain a practical arrangement of the steel at internal supports.

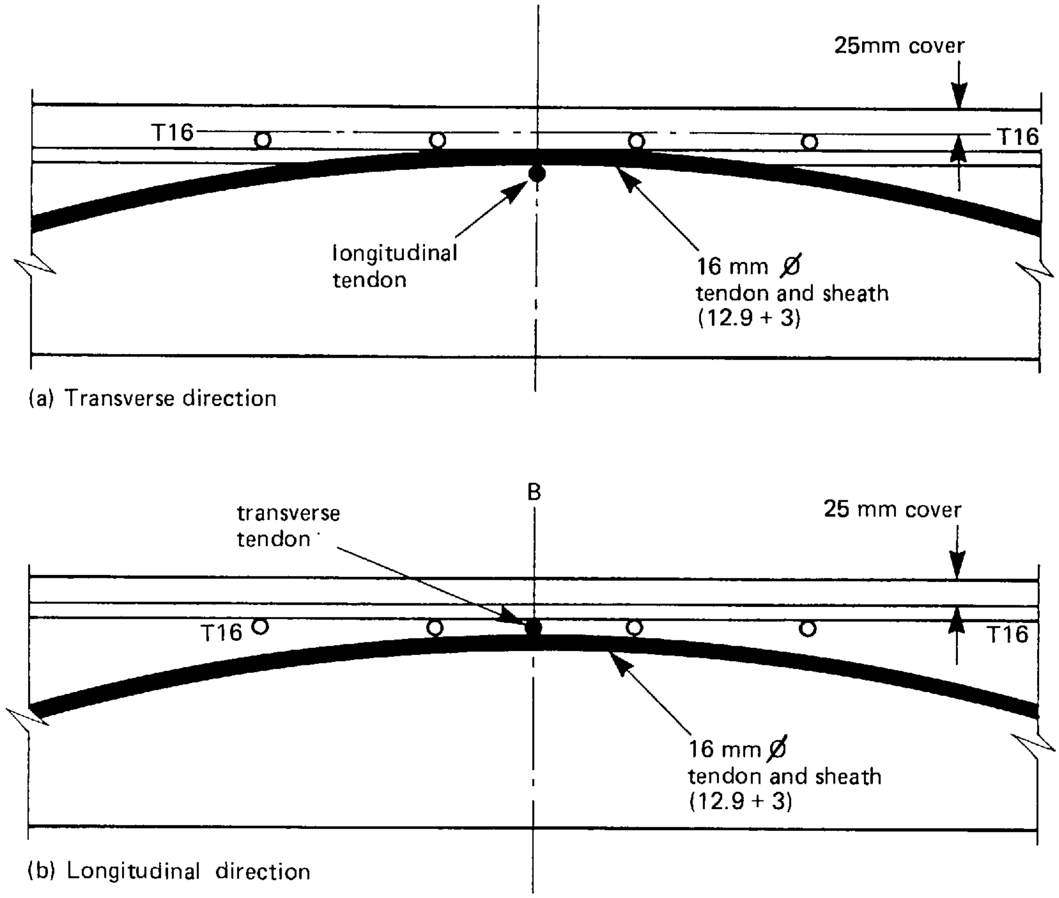


Figure A2: Tendon and reinforcing steel positioning

Preliminary shear check

Taking a slab depth of 225 mm, check the punching shear capacity, and the shear capacity at the face of the column, for both internal and external columns.

Internal Columns (500 × 500)

The load from the slab on to the internal columns will be greater than that due to half the span because of elastic distribution (see 6.6). For this example, a reasonable estimate for the increase in equivalent floor area is a factor of 1.2.

Equivalent floor area = 1.2 (3.5 + 2.25) × 7 = 48.3 m^2

From Figure 17b for a total imposed load of 7.2 kN/m^2 and equivalent floor area of 48.3 m^2, some shear reinforcement will be required for a slab of depth 225 mm.
From Figure 18 the check for maximum shear stress ($0.8\sqrt{f_{cu}}$ or 5 N/mm^2) is fine.

Edge Columns (300 × 300)

Figures 17 and 18 are set up for internal columns. In order to use the figures for an edge column, the equivalent loaded area is doubled.

Floor area = 2 × 3.5 × 3.5 = 24.5 m^2

From Figure 17a for a total imposed load of 7.2 kN/m^2 and equivalent floor area of 24.5 m^2, no shear reinforcement is required for a slab depth of 225 mm.
From Figure 18 the check for maximum shear stress ($0.8\sqrt{f_{cu}}$ or 5 N/mm^2) is fine.

A1.1 *Serviceability Limit State*

A1.1.1 Transverse direction

After deciding the limiting tendon eccentricities (Figure A2) and the positions of the points of inflection - 0.1 times the span, from the centre of supports - the tendon profile can be calculated; see Appendix C.

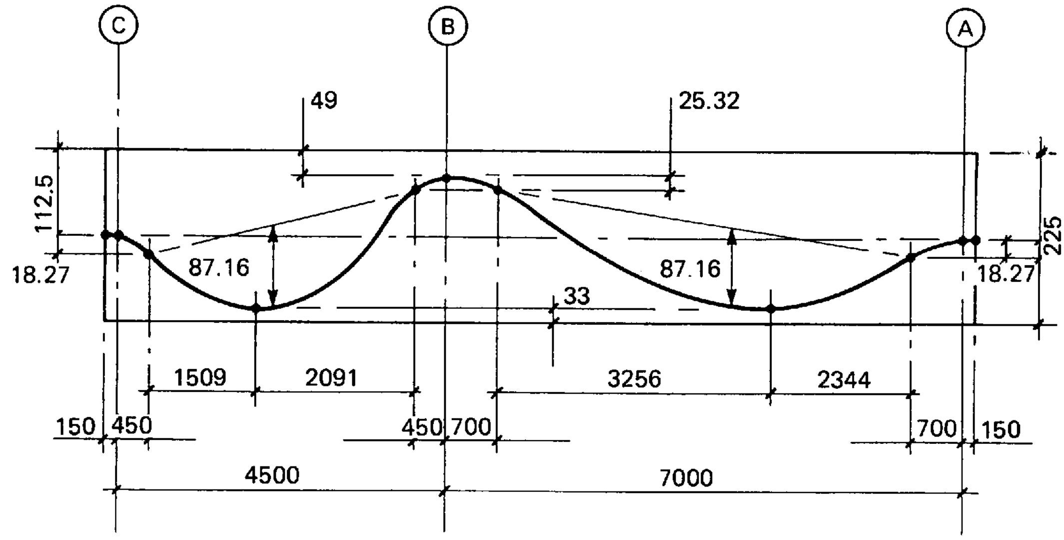


Figure A3: Transverse tendon profile

Calculation of maximum drape

Assume that the maximum drape occurs at midspan. Using the equation of a parabola:

$$y = kx(s-x)$$

from the tendon profile calculation (Appendix C), we know that:

$$k = 2.69 \times 10^{-5}$$

also $s = 3600$ mm

when $x = 1800$ mm $y = 87.16$ mm

At this stage losses are assumed as follows:

At transfer 10% of the jacking load

At service 20% of the jacking load.

A thorough check will be carried out after the stress calculations to check that these initial assumptions of 10% and 20% are within reason. If they are not, another estimate should be made and the procedure repeated.

Initial prestress

The initial prestress force, i.e. the jacking force, has been taken to be 70% of the characteristic strength (see BS8110, Part 1, Cl 4.7).

For the transverse direction, the tendons will be stressed along gridline A only.

Calculation of P_{av}

Jacking force = 0.7 × 186	=	130.20 kN/tendon
Prestress force at transfer (10% losses)	=	117.18 kN/tendon
Prestress force at service (20% losses)	=	104.16 kN/tendon

Next the value of prestress force required in each span is calculated. This is done using the chosen balanced load of 8.6 kN/m^2 (the dead load), the distance between points of inflection, s, and the drape a, as shown in Figure A4.

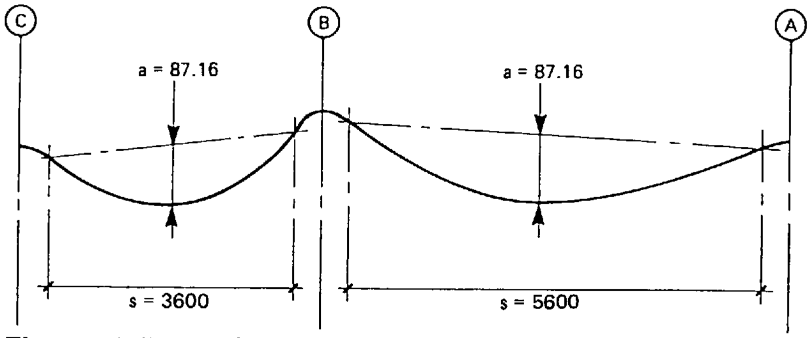


Figure A4: Drapes for load balancing

The prestress force is obtained from the following equation, which assumes a parabolic profile.

$$P_{reqd} = \frac{ws^2}{8a}$$

For span CB, $$P_{reqd} = \frac{8.6 \times 7 \times 3600^2}{8 \times 87.16 \times 1000} = 1119 \text{ kN}$$

Therefore number of tendons $$= \frac{1119}{104.16} = 10.7;$$ try 11 tendons per panel

For span BA, $$P_{reqd} = \frac{8.6 \times 7 \times 5600^2}{8 \times 87.16 \times 1000} = 2707 \text{ kN}$$

Therefore number of tendons $$= \frac{2707}{104.16} = 25.99;$$ try 26 tendons per panel

As the longer span requires more tendons than the shorter span, 15 of the tendons will be stopped off at the point of inflection in span CB, next to support B. When accurate losses are calculated, the different force profile of these shorter tendons must be taken into account.

The effect of the tendons on the slab is modelled by means of equivalent loads, as shown below. Equivalent loads are discussed in more detail in Appendix D and Section 6.9 of this report. It should be noted that the portions of the cable from the edges of the slab to gridlines A and C are horizontal and so do not contribute to the equivalent loads.

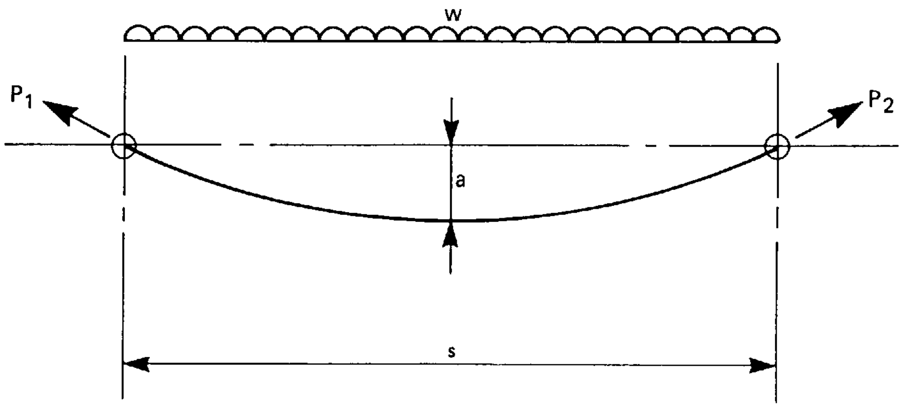


Figure A5: Calculation of equivalent loads due to tendon

The equivalent load, w, between any two points of inflection for the chosen number of tendons is given by:

$$w = \frac{8anP_{av}}{s^2}$$

where:

n is the number of tendons
a is the drape at the point considered
s is as shown in Figure A5
P_{av} is the average force provided by each tendon.

C B A

Equivalent loads at transfer						
Full length tendons (n = 11)						
$n \times P_{av}$(kN)	1289.0	1289.0	1289.0	1289.0	1289.0	1289.0
a (mm)	18.3	-87.2	25.3	25.3	-87.2	18.3
s (mm)	900	3600	900	1400	5600	1400
w (kN/m)	232.6	-69.4	322.3	133.2	-28.7	96.1
Short tendons (n = 15)						
$n \times P_{av}$(kN)	/	/	1757.7	1757.7	1757.7	1757.7
a (mm)	/	/	25.3	25.3	-87.2	18.3
a (mm)	/	/	900	1400	5600	1400
w (kN/m)	/	/	439.6	181.7	-39.1	131.1
Total w (kN/m)	232.6	-69.4	761.9	314.9	-67.7	227.2

C B A

Equivalent loads at transfer						
Full length tendons (n = 11)						
$n \times P_{av}$(kN)	1145.8	1145.8	1145.8	1145.8	1145.8	1145.8
a (mm)	18.3	-87.2	25.3	25.3	-87.2	18.3
s (mm)	900	3600	900	1400	5600	1400
w (kN/m)	206.8	-61.6	286.5	118.4	-25.5	85.4
Short tendons (n = 15)						
$n \times P_{av}$(kN)	/	/	1562.4	1562.4	1562.4	1562.4
a (mm)	/	/	25.3	25.3	-87.2	18.3
a (mm)	/	/	900	1400	5600	1400
w (kN/m)	/	/	390.7	161.5	-34.7	116.5
Total w (kN/m)	206.8	-61.6	383.3	158.4	-60.2	202.0

Table A1: Calculations of equivalent loads due to transverse tendons, at transfer and after all losses.

When tendons are anchored within the span, as in this example, additional equivalent loads may be generated by the end condition. These must be included in the frame analysis when obtaining the bending moments and shear force diagrams. The forces consist of a vertical and horizontal component of the tendon force applied at the anchor.

Figure A6 below shows the effect of an anchorage in terms of additional equivalent loads on the slab.

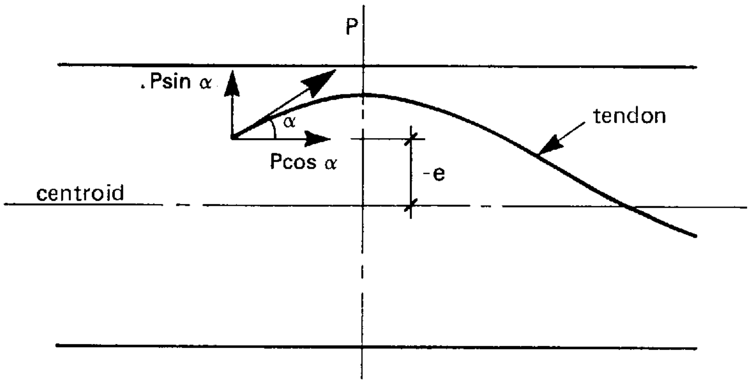


Figure A6: Equivalent loads at anchorages.

The vertical component of the tendon force is easily calculated, and should be applied to the slab as a vertical point load at the point where the tendon is anchored. The horizontal component forms a positive moment about the centroid, owing to its eccentricity from the centroid of the section, and should be applied in this form to the slab.

It should be noted that the position of the tendon at the anchorage can be arranged so that the tendon is both horizontal (no vertical force) and at the centroid of the section (no eccentric moment). In this example the anchorages at the ends of the full-length tendons fulfil this requirement and no additional loads are generated.

Vertical force = $P\sin\alpha$

Eccentric moment at the point of inflection = $P\cos\alpha \times e$

For a parabolic tendon $\dfrac{dy}{dx} = \dfrac{2ax}{s^2}$

and so $\sin\alpha \approx \dfrac{2a}{s}$

Therefore:

$$\sin\alpha = \frac{2 \times 25.32}{450} = 0.1125$$

$$\cos\alpha = 0.9937$$

Eccentricity of tendon = -112.5 + 25(cover) + 16(diameter of un-tensioned reinforcement) + 8(half the tendon diameter) + 25.32(drape)

= -38.18 mm

At transfer

$P = 1757.70$ kN

$P\sin\alpha = 197.82$ kN

$Pe\cos\alpha = -66.682$ kNm

After all losses

$P = 1562.4$ kN

$P\sin\alpha = 175.84$ kN

$Pe\cos\alpha = -59.276$ kNm

The equivalent loads from the tendons, the anchors and the superimposed loads are then used to calculate design moments and shears by any convenient method of structural analysis. This is normally done using an appropriate computer program.

At Seviceability Limit State, an elastic method of analysis should be used for analysing post-tensioned flat slabs, and patterned loading should be used in multi-span situations (see BS8110, Part 1, Section 4.4[4]).

Summary of prestress equivalent loads

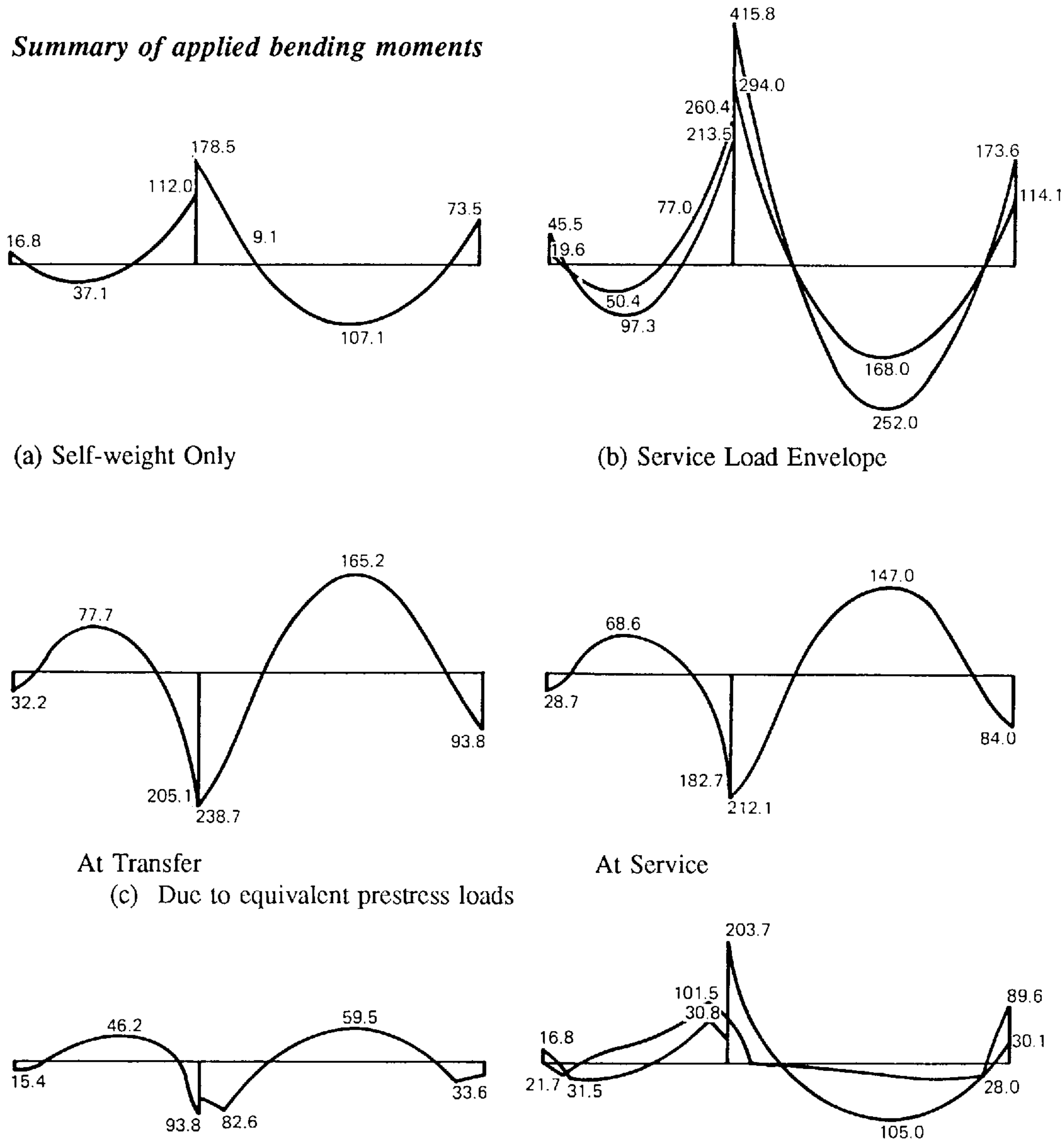
	C		B			A
Equivalent loads at transfer (kN/m)	232.6	-69.4	761.9	314.9	-67.7	227.2
Equivalent loads after all losses (kN/m)	206.8	-61.6	677.3	279.9	-60.2	202.0

Table A2: Summary of uniformly distributed equivalent loads from transverse tendons.

	Vertical force	Eccentric moment
At transfer	197.8 kN	66.7 kNm
At service	175.8 kN	59.3 kNm

Table A3: Summary of additional equivalent loads due to internal anchorages.

Summary of applied bending moments



(a) Self-weight Only

(b) Service Load Envelope

At Transfer

At Service

(c) Due to equivalent prestress loads

At Transfer

At Service

(d) Total Applied Load

Figure A7: Applied bending moment diagrams

Check that Prestress Loads total zero

Upward loads $= 61.6 \times 3.6 + 60.2 \times 5.6 + 175.84$

$= 734.72$ kN

Downward loads $= (206.8 + 677.3) \times 0.45 + (279.9 + 202.0) \times 0.7$

$= 735.18$ kN

The small difference between these values is due to earlier approximations. The equivalent loads were altered to total zero at this point to enable consistent calculation of secondary moments.

Calculation of stresses

$$f_t = \frac{P}{A_c} - \frac{Pe}{z_t} + \frac{M_A}{z_t} + \frac{M_s}{z_t}$$

$$f_b = \frac{P}{A_c} + \frac{Pe}{z_b} - \frac{M_A}{z_b} - \frac{M_s}{z_b}$$

$$A_c = 7 \times 0.225 \times 10^6 = 1.575 \times 10^6 \text{ mm}^2$$

As the section being considered is rectangular and symmetrical about the centroid, z_t and z_b are equal.

$$z_t = z_b \equiv z = \frac{bh^2}{6} = 5.91 \times 10^7 \text{ mm}^3$$

As this example is a flat slab, analysed by the equivalent frame method, the allowable stresses are as detailed in Table 2 (Section 6.10.1) of this report. To increase ease of construction, untensioned reinforcement has been deliberately omitted from the spans by keeping tensile stresses below $0.15\sqrt{f_{ci}}$ (transfer) and $0.15\sqrt{f_{cu}}$ (service).

All tensile stresses are negative.

Zone		Stress due to prestess * (N/mm^2)	Stress due to self-weight (N/mm^2)	Total stress (N/mm^2)	Allowable stress (N/mm^2)
C	top	1.364	-0.280	1.084	6.00
	bottom	0.272	0.280	0.552	-2.25
CB (hogging)	top	-0.492	0.534	0.042	-0.75
	bottom	2.128	-0.534	1.594	8.25
B	top	5.406	-1.891	3.515	6.00
	bottom	-1.538	1.891	0.353	-2.25
B	top	4.755	-1.429	3.326	6.00
	bottom	-0.887	1.429	0.542	-2.25
BA (sagging)	top	2.509	-0.154	2.355	8.25
	bottom	1.359	0.154	1.513	-0.75
BA (hogging)	top	-0.862	1.790	0.928	-0.75
	bottom	4.730	-1.790	2.940	8.25
A	top	2.504	-0.006	2.498	6.00
	bottom	1.364	0.006	1.370	-2.25

Table A4: Stresses at transfer for the transverse direction

* These values include prestress secondary effects.

Zone		Stress due to prestess * (N/mm^2)	Stress due to service-load (N/mm^2)	Total stress (N/mm^2)	Allowable stress (N/mm^2)
C	top	1.27	-0.77	0.50	-2.85
	bottom	0.18	0.77	0.95	9.60
CB (sagging)	top	-0.23	1.49	1.26	13.20
	bottom	1.68	-1.49	0.19	-0.95
CB (hogging)	top	0.83	-1.31	-0.48	-0.95
	bottom	0.62	1.31	1.93	13.20
B	top	2.67	-2.67	0.00	-2.85
	bottom	0.77	2.67	3.44	9.60
B	top	5.31	-7.04	-1.73	-2.85
	bottom	-1.88	7.04	5.16	9.60
BA (sagging)	top	-0.77	4.27	3.50	13.20
	bottom	4.21	-4.27	-0.06	-0.95
A	top	3.13	-2.93	0.20	-2.85
	bottom	0.30	2.93	2.23	9.60

Table A5: Stresses after all losses for the transverse direction.

Hogging and sagging values are given where they are both in one zone. Each span is split into three zones, from the end to $\frac{2L}{10}$, from $\frac{2L}{10}$ to $\frac{8L}{10}$ and from $\frac{8L}{10}$ to L.

In this example, the construction load is smaller than the load at service and larger than that at transfer. This means that the construction case is not likely to be a governing situation and so the stresses are not calculated.

Loss calculations

At this stage the losses should be calculated accurately to check that the initial assumptions of 10% at transfer and 20% at service were reasonable. The method for calculating the various types of loss is given in Appendix B.

Full-length tendons

Short-term losses

a) Losses due to friction

$$P_x = P_o \times e^{-\mu x(\alpha' + \varpi)}$$

Table B1 gives recommended values for the coefficients μ and ϖ:

$\mu = 0.06$ and $\varpi = 0.05$ rads/m

deviated angle per metre, $\alpha' = \dfrac{16 \times \text{total drape}}{L^2}$

total drape (the same for both spans) $= \dfrac{18.27 + 25.32}{2} + 87.16 = 108.96$ mm

Span CB

$$\alpha' = \frac{16 \times 108.96 \times 10^{-3}}{4.5^2} = 0.086 \text{ rad/m}$$

Span BA

$$\alpha' = \frac{16 \times 108.96 \times 10^{-3}}{7^2} = 0.036 \text{ rad/m}$$

Jacking force = 130.2 kN

Forces after friction losses (see Figure A8) are:

$P_A = 130.2$ kN

$P_B = 130.2 \times e^{-7\times0.06(0.036+0.05)}$ = 125.6 kN

$P_C = 125.58 \times e^{-4.5\times0.06(0.086+0.05)}$ = 121.1 kN

b) Losses due to wedge set

force loss at anchorage, $\delta P_w = 2p'l'$

where: p' = slope of force profile
l' = length of tendon effected by draw-in

$l' = \sqrt{((\Delta \times E_{ps} \times A_{ps})/p')}$

take wedge draw-in,
Δ = 6 mm
Eps = 195 kN/mm^2
Aps = 100 mm^2

$$p' = \frac{P_A - P_C}{L_1 + L_2}$$

$$p' = \frac{130.2 - 121.07}{7 + 4.5} = 0.79 \text{ kN/m}$$

therefore, $l' = \sqrt{((6 \times 10^{-3} \times 195 \times 100)/0.79)}$

$l' = 12.14$ m

As l' is greater than the length of the tendon,

δP_w at stressing anchorage $= (\Delta \times E_{ps} \times A_{ps})/l + (p' \times l)$

and

δP_w at dead end $= (\Delta \times E_{ps} \times A_{ps})/l - (p' \times l)$

therefore,

δP_w at stressing anchorage $= \dfrac{(6 \times 195 \times 100)}{11.5 \times 10^3} + 0.79 \times 11.5 = \underline{19.30 \text{ kN}}$

δP_w at dead end $= \dfrac{(6 \times 195 \times 100)}{11.5 \times 10^3} - 0.79 \times 11.5 = \underline{1.04 \text{ kN}}$

Forces after friction losses and wedge set: (see Figure A8)

$P_A = 130.2 - 19.30 = 110.9$ kN

$P_B = 125.58 - (19.30 - 1.04) \times \dfrac{4.5}{11.5} - 1.04 = 117.4$ kN

$P_C = 121.07 - 1.04 = 120.0$ kN

c) Elastic losses

$$\delta P_{es} = \varepsilon_{es} \times E_{ps} \times A_{ps}$$

where: $\varepsilon_{es} = 0.5 \times \dfrac{f_{co}}{E_{ci}}$

f_{co} is the stress in the concrete adjacent to the tendon. Since this is unlikely to be critical, the stress is calculated at a representative point and will be taken as uniform over the whole tendon length.

$f_{co} = 1.984$ N/mm^2

$\varepsilon_{es} = 0.5 \times \dfrac{1.984}{21.7 \times 10^3} = 4.57 \times 10^{-5}$

$\delta P_{es} = 4.57 \times 10^{-5} \times 195 \times 100 = \underline{0.89 \text{ kN}}$

Prestress at transfer

Prestress force at A = 110.9 - 0.89 = 110.0 kN
Prestress force at B = 117.4 - 0.89 = 116.5 kN
Prestress force at C = 120.0 - 0.89 = 119.1 kN

Long-term losses

a) Relaxation of steel

$\delta P_r =$ 1000-hour relaxation value $\times$ relaxation factor $\times$ prestress force at transfer

From Table B2 in Appendix B values are taken for an initial jacking force equal to 70% of the characteristic strength.

loss due to relaxation = 2.5%

relaxation factor = 1.5

therefore, $\delta P_r = 2.5\% \times 1.5 \times P_x = 3.75\% \times P_x$

$\delta PrA = 0.0375 \times 110.01 = 4.13$ kN
$\delta P_{rB} = 0.0375 \times 116.50 = 4.37$ kN
$\delta P_{rC} = 0.0375 \times 119.14 = 4.47$ kN

b) Shrinkage of concrete

$\delta P_{sh} = \varepsilon_{sh} \times E_{ps} \times A_{ps}$

from BS 8110, Part 1, Clause 4.8.4[4]

$\varepsilon_{sh} = 300 \times 10^{-6}$

$\delta P_{sh} = 300 \times 10^{-6} \times 195 \times 100 = \underline{5.85 \text{ kN}}$

c) Creep of concrete

$\delta P_{cr} = \varepsilon_{cc} \times E_{ps} \times A_{ps}$

$$\varepsilon_{cc} = \frac{f_{co}}{E_{ci}} \times \phi$$

where ϕ = creep coefficient (see Appendix B)

$\phi = 2.0$

$f_{co} = 1.984$ N/mm^2

$$\varepsilon_{cc} = \frac{1.984}{21.7 \times 10^3} \times 2$$

$= 1.83 \times 10^{-4}$

$\delta P_{cr} = 1.83 \times 10^{-4} \times 195 \times 100 = \underline{3.57 \text{ kN}}$

Prestress after all losses

Prestress force at A = 110.0 - 4.13 - 5.85 - 3.57 = 96.5 kN
Prestress force at B = 116.5 - 4.37 - 5.85 - 3.57 = 102.7 kN
Prestress force at C = 119.1 - 4.47 - 5.85 - 3.57 = 105.3 kN

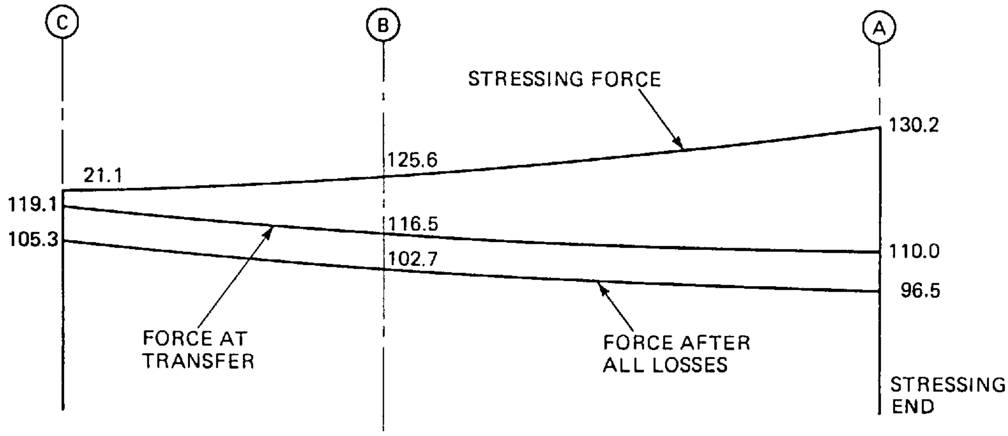


Figure A8: Force profiles for full-length tendons

- Short tendons

Friction losses and elastic losses are the same as for the full-length tendons, as are the long-term losses. The effect of wedge set is different as the tendon length is different and must be recalculated.

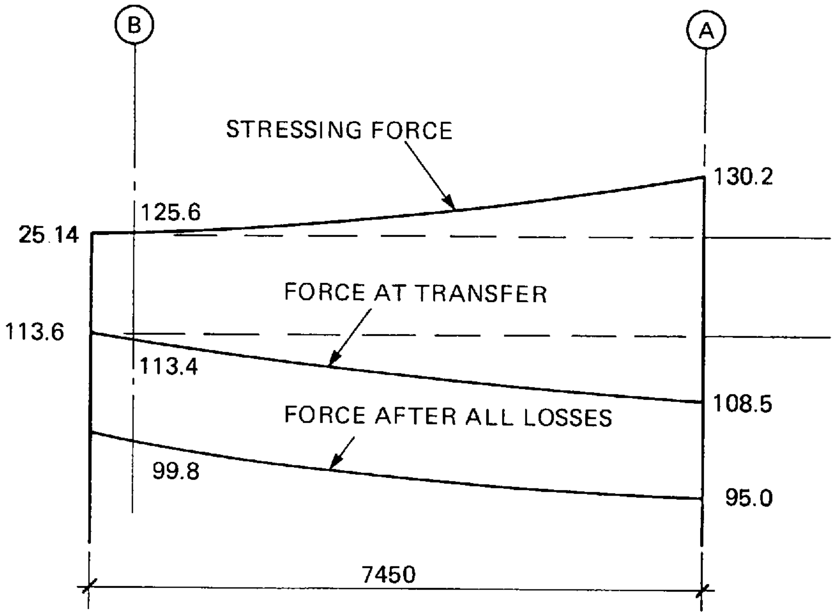


Figure A9: Force profile for short tendons

Force at dead end = $125.58e^{-0.45 \times 0.06(0.086 + 0.05)}$ = 125.14 kN

δP_w at dead end = $(\Delta \times E_{ps} \times A_{ps})/l - (p' \times l)$

$$= 6 \times 195 \times 100/7450 - \frac{(130.2 - 125.14)}{7450} \times 7450 \quad = 10.64 \text{ kN}$$

δP_w at stressing anchorage $= (\Delta \times E_{ps} \times A_{ps})/l + (p' \times l)$

$$= 6 \times 195 \times 100/7450 + \frac{(130.2 - 125.14)}{7450} \times 7450 = 20.77 \text{ kN}$$

Forces after friction losses and wedge set:

$$P_A = 130.2 - 20.77 = 109.4 \text{ kN}$$
$$P_B = 125.58 - \frac{45}{745}(20.77 - 10.64) - 10.64 = 114.3 \text{ kN}$$

Prestress at transfer

Prestress force at A = 109.4 - 0.89 = 108.5
Prestress force at B = 114.3 - 0.89 = 113.4

Prestress after all losses

Prestress force at A = 108.5 - 4.07 - 5.85 - 3.57 = 95.0 kN
Prestress force at B = 113.4 - 4.25 - 5.85 - 3.57 = 99.8 kN

Short- and long-term losses

Check that the losses assumed initially are reasonable by calculating accurately.

Average short-term loss for span CB
(full-length tendons only)

$$= (8.5\% + 10.5\%)/2 = \underline{9.5\%}$$

Average short-term loss for span BA
(both tendon lengths)

$$= \frac{11}{26} \times \frac{(10.5\% + 15.5\%)}{2} + \frac{15}{26} \times \frac{(12.9\% + 16.6\%)}{2} = \underline{14.0\%}$$

Average overall short-term loss = <u>11.8%</u>

Average long-term loss for span CB
(full-length tendons only)

$$= (19.2\% + 21.1\%)/2 = \underline{20.2\%}$$

Average long-term loss for span BA
(both tendon lengths)

$$= \frac{11}{26} \times \frac{(21.1\% + 25.9\%)}{2} + \frac{15}{26} \times \frac{(23.4\% + 27.0\%)}{2} = \underline{24.5\%}$$

Average overall long-term loss = <u>22.3%</u>

Although the assumed losses of 10% and 20%, respectively, have been exceeded in span BA, recalculation is not considered necessary, as it will not cause an increase in the number of tendons. Also the calculation of stresses for the correct losses are unlikely to exceed the allowable values.

A1.1.2 *Longitudinal direction*

Analysis of longitudinal frame along gridline B.

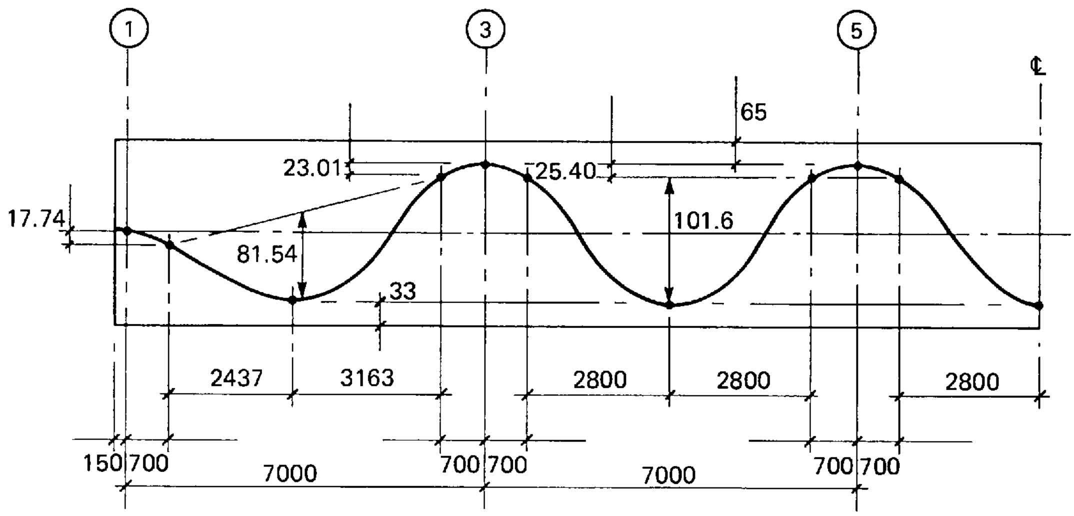


Figure A10: Longitudinal tendon profile

The method of calculation follows that given for the transverse section.

Losses as assumed before,

10% of the jacking load at transfer
20% of the jacking load at service

Initial prestress forces as before

Prestress force at transfer, P_{av}	= 117.18 kN/tendon
Prestress force at service, P_{av}	= 104.16 kN/tendon

The elastic reaction on the internal columns along grid line B at working load was calculated for the transverse direction,

$$R_{max} = 593 \text{ kN}$$

The total uniformly distributed load = 12.6 kN/m²

Hence the effective width of slab $= \dfrac{593}{12.6 \times 7}$ = 6.72 m

Assume a balanced load equal to 100% dead load as for the transverse direction.

$$\text{Required } P = \frac{ws^2}{8a}$$

Where $w = 8.6 \text{ kN/m}^2$
$s = 5600 \text{ mm}$
$a = 81.54 \text{ mm}$ (for span 1 to 3 and 9 to 11)
101.6 mm (for all other spans)

The length of slab is greater than 30m and so stressing is assumed to take place from both ends. This produces symmetry about gridline 6 and so the tendon requirements on either side of this will be the same.

Span 1 - 3 : $P_{reqd} = \dfrac{8.6 \times 5600^2}{8 \times 81.54 \times 1000} = 413.44$ kN/m width

Span 3 - 5,5 - 7 : $P_{reqd} = \dfrac{8.6 \times 5600^2}{8 \times 101.6 \times 1000} = 331.81$ kN/m width

Choose number of tendons for this load balance:

Span 1 - 3 : No. of tendons $= \dfrac{413.44 \times 6.72}{104.16} = 26.7$, say 27

Span 3 - 5,5 - 7 : No. of tendons $= \dfrac{331.81 \times 6.72}{104.16} = 21.4$, say 22

1 3 5 6

Equivalent loads at transfer							
Full-length tendons (n = 22)							
$n \times P_{av}$(kN)	2578	2578	2578	2578	2578	2578	2578
a (mm)	17.7	-81.5	23.0	25.4	-101.6	25.4	-101.6
s (mm)	1400	5600	1400	1400	5600	1400	5600
w (kN/m)	186.7	-53.6	242.1	267.3	-66.8	267.3	-66.8
Short tendons (n = 5)							
$n \times P_{av}$(kN)	586	586	586	586	/	/	/
a (mm)	17.7	-81.5	23.0	25.4	/	/	/
a (mm)	1400	5600	1400	1400	/	/	/
w (kN/m)	42.4	-12.2	55.0	60.7	/	/	/
Total w (kN/m)	229.1	-65.8	297.1	328.0	-66.8	267.3	-66.8

1 3 5 6

Equivalent loads after all losses							
Full-length tendons (n = 22)							
$n \times P_{av}$(kN)	2291.5	2291.5	2291.5	2291.5	2291.5	2291.5	2291.5
a (mm)	17.7	-81.5	23.0	25.4	-101.6	25.4	-101.6
s (mm)	1400	5600	1400	1400	5600	1400	5600
w (kN/m)	165.9	-47.7	215.2	237.6	-59.4	237.6	-59.4
Short tendons (n = 5)							
$n \times P_{av}$(kN)	521	521	521	521	/	/	/
a (mm)	17.7	-81.5	23.0	25.4	/	/	/
a (mm)	1400	5600	1400	1400	/	/	/
w (kN/m)	37.7	-10.8	48.9	54.0	/	/	/
Total w (kN/m)	203.6	-58.5	264.1	291.6	-59.4	237.6	-59.4

Table A6: Calculations of equivalent loads due to longitudinal tendons, at transfer and after all losses.

Effect of tendons anchored within the span

$$\sin\alpha = \frac{2 \times 25.4}{700} = 0.073$$

$$\cos\alpha = 0.997$$

Eccentricity of tendon $= 112.5 - 25 - 32 - 8 - 25.4 = 22.10$ mm

At transfer

$P = 586$ kN

$P\sin\alpha = 42.527$ kN

$Pe\cos\alpha = 12.916$ kNm

After all losses

$P = 521$ kN

$P\sin\alpha = 37.810$ kN

$Pe\cos\alpha = 11.484$ kNm

Summary of prestress equivalent load analysis

1 3 5 6

Equivalent loads at transfer (kN/m)	229.1	-65.8	297.1	328.0	-66.8	267.3	-66.8
Equivalent loads after all losses (kN/m)	203.6	-58.5	264.1	291.6	-59.4	237.6	-59.4

Table A7: Summary of uniformly distributed equivalent loads from longitudinal tendons.

	Vertical force	Eccentric moment
At transfer	42.5 kN	12.9 kNm
At service	37.8 kN	11.5 kNm

Table A8: Summary of additional forces due to internal anchorages

Summary of applied bending moments

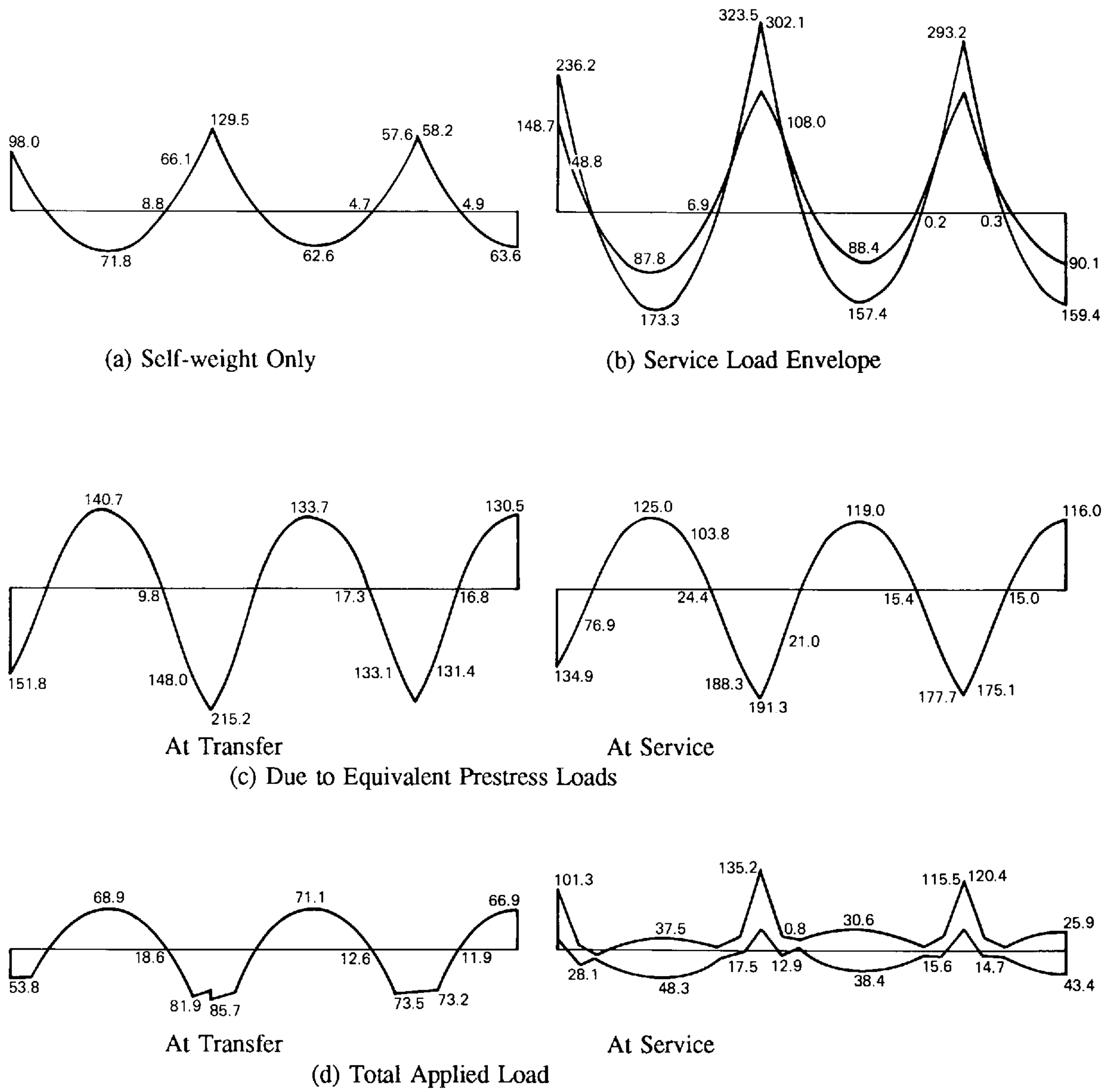


Figure A11: Applied bending moment diagrams

Calculation of stresses

Stresses are calculated as for the transverse direction.

Zone		Stress due to prestress * (N/mm^2)	Stress due to self-weight (N/mm^2)	Combined stress (N/mm^2)	Allowable stress (N/mm^2)
1	top	4.61	-1.39	3.22	6.00
	bottom	0.28	1.39	1.67	-2.25
1-3 (hogging)	top	-1.36	2.00	0.64	-0.75
	bottom	6.25	-2.00	4.25	8.25
1-3 (sagging)	top	3.32	-0.36	2.96	8.25
	bottom	1.57	0.36	1.93	-0.75
3	top	6.43	-1.87	4.56	6.00
	bottom	-1.54	1.87	0.34	-2.25
3	top	7.75	-3.19	4.56	6.00
	bottom	-2.86	3.19	0.33	-2.25
3-5 (hogging)	top	-1.18	1.48	0.30	-0.75
	bottom	5.17	-1.48	3.69	8.25
3-5 (sagging)	top	2.39	-0.10	2.29	8.25
	bottom	1.59	0.10	1.69	-0.75
5	top	5.16	-1.36	3.80	6.00
	bottom	-1.17	1.36	0.19	-2.25
5	top	5.14	-1.39	3.75	6.00
	bottom	-1.16	1.39	0.23	-2.25
5-7 (sagging)	top	2.39	-0.12	2.27	8.25
	bottom	1.60	0.12	1.72	-0.75
5-7 (hogging)	top	-1.15	1.53	0.38	-0.75
	bottom	5.14	-1.53	3.61	8.25

Table A9: Stresses at transfer for the longitudinal direction

Zone		Stress due to prestress * (N/mm^2)	Stress due to service load (N/mm^2)	Combined stress (N/mm^2)	Allowable stress (N/mm^2)
1	top	4.09	-3.34	0.75	-2.85
	bottom	0.25	3.34	3.59	9.60
1-3 (hogging)	top	-1.21	4.88	3.67	13.20
	bottom	5.56	-4.88	0.68	-0.95
1-3 (sagging)	top	-0.51	2.32	1.81	-0.95
	bottom	4.86	-2.32	2.53	13.20
3	top	7.26	-8.76	-1.50	-2.85
	bottom	-2.91	8.76	5.85	9.60
3	top	6.89	-7.44	-0.55	-2.85
	bottom	-2.54	7.44	4.90	9.60
3-5 (hogging)	top	-1.05	2.10	1.05	-0.95
	bottom	4.59	-2.10	2.49	13.20
3-5 (sagging)	top	-1.05	3.72	2.67	13.20
	bottom	4.59	-3.72	0.87	-0.95
5	top	5.99	6.97	-0.98	-2.85
	bottom	-2.45	6.97	4.52	9.60
5	top	5.97	-7.09	-1.12	-2.85
	bottom	-2.43	7.09	4.66	9.60
5-7 (sagging)	top	-1.02	2.209	1.18	-0.95
	bottom	4.56	-2.209	2.36	13.20
5-7 (hogging)	top	-1.02	3.81	2.79	13.20
	bottom	4.56	-3.81	0.75	-0.95

Table A10: Stresses after all losses for the longitudinal direction

Loss calculations

Losses are now calculated as before to ensure that the initial assumptions are reasonable.

- Full-length tendons

Short-term losses

a) losses due to friction

Span 1-3: Total drape $= 101.92$ mm

$$\alpha' = \frac{16 \times 101.92}{7^2 \times 1000} = 0.033 \text{ rad/m}$$

Span 3-5: Total drape $= 127$ mm

$$\alpha' = \frac{16 \times 127.00}{7^2 \times 1000} = 0.041 \text{ rad/m}$$

Span 5-6: Total drape $= 127$ mm

$\alpha' = 0.041$ rad/m

Forces after friction losses are:

$$P_1 = 0.7 \times 186 = 130.2 \text{ kN}$$

$$P_3 = 130.2e^{-7 \times 0.06(0.033+0.05)} = 125.7 \text{ kN}$$

$$P_5 = 125.72e^{-7 \times 0.06(0.041+0.05)} = 121.0 \text{ kN}$$

$$P_6 = 120.99e^{-3.5 \times 0.06(0.041+0.05)} = 118.7 \text{ kN}$$

b) losses due to wedge set

$p' = 0.658$ kN/m

$l' = 13.33$ m

l' is less than the tendon length, therefore

$$\delta P_w = 2p'l'$$

$$\delta P_w = 2 \times 0.66 \times 13.33 = \underline{17.55 \text{ kN}}$$

Forces after friction loss and wedge set

$$P_1 = 130.2 - 17.55 = 112.7 \text{ kN}$$

$$P_3 = 125.7 - \frac{17.55 \times 6.33}{13.33} = 117.4 \text{ kN}$$

$$P_5 = 121.0 \text{ kN}$$

$$P_6 = 118.7 \text{ kN}$$

c) elastic losses

$f_{co} \approx 2.67$ N/mm^2

$\varepsilon_{es} = 6.15 \times 10^{-5}$

$\delta P_{es} = 1.20$ kN

Prestress at transfer

(See Figure A12)

Prestress force at 1	= 112.6 - 1.20	=	111.5 kN
Prestress force at 3	= 117.4 - 1.20	=	116.2 kN
Prestress force at 5	= 121.0 - 1.20	=	119.8 kN
Prestress force at 6	= 118.7 - 1.20	=	117.5 kN

Long-term losses

a) relaxation of steel

as before <u>3.75%</u>

$\delta P_{r1} = 4.18$ kN
$\delta P_{r2} = 4.36$ kN
$\delta P_{r3} = 4.49$ kN
$\delta P_{r4} = 4.41$ kN

b) shrinkage of concrete

as before <u>5.85kN</u>

c) creep of concrete

$f_{co} = 2.67$ N/mm^2

$\varepsilon_{cc} = 2.46 \times 10^{-4}$

$\delta P_{cr} = \underline{4.80 \text{ kN}}$

Prestress force after all losses

Prestress force at 1	= 111.5 - 4.18 - 5.85 - 4.8	=	96.6 kN
Prestress force at 3	= 116.2 - 4.36 - 5.85 - 4.8	=	101.2 kN
Prestress force at 5	= 119.8 - 4.49 - 5.85 - 4.8	=	104.7 kN
Prestress force at 6	= 117.5 - 4.41 - 5.85 - 4.8	=	102.4 kN

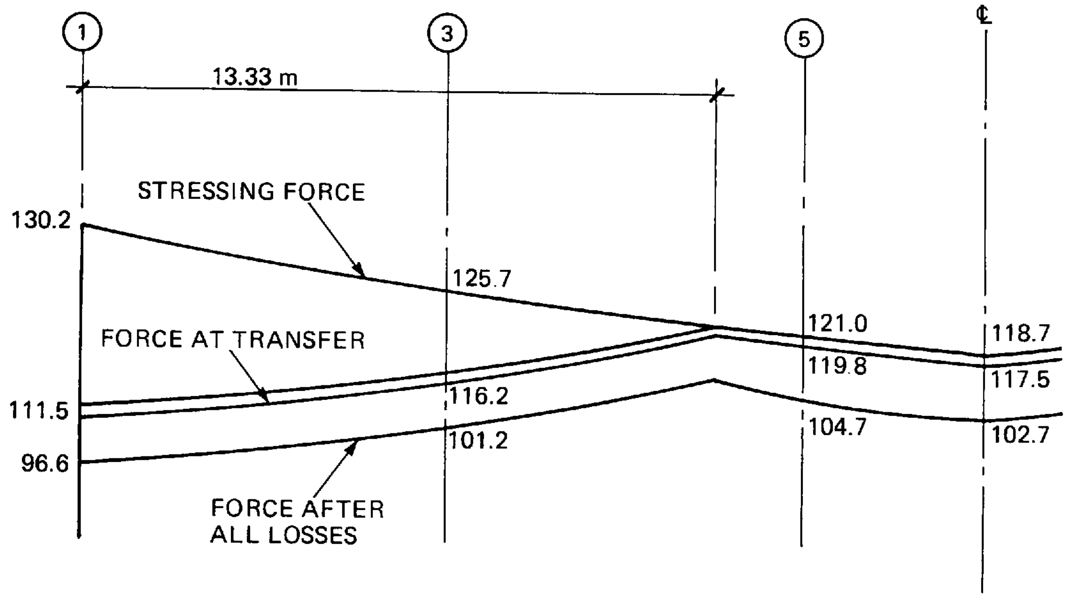


Figure A12: Force profile for full-length longitudinal tendons.

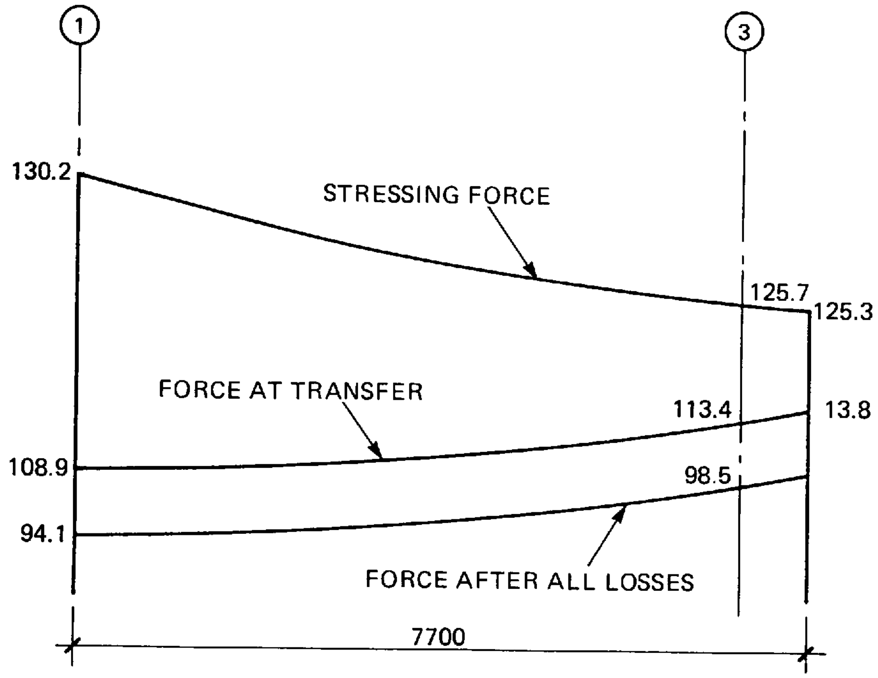


Figure A13: Force profile for short longitudinal tendons.

- Short tendons

Recalculate wedge set for the shorter tendons, all other losses as before.

Force at dead end $= 125.72e^{-0.7 \times 0.06(0.041 + 0.05)}$ $=$ 125.25 kN

δP_w at dead end $= 6 \times 195 \times 100/7700 - \dfrac{(130.2 - 125.25) \times 7700}{7700}$

$=$ 10.24 kN

δP_w at stressing anchorage $= 6 \times 195 \times 100/7700 + \dfrac{(130.2 - 125.25) \times 7700}{7700}$

$=$ 20.14 kN

Forces after friction losses and wedge set:

$P_1 = 130.2 - 20.14 = 110.1$ kN

$P_3 = 125.7 - \dfrac{7}{77}(20.14 - 10.24) - 10.24$ $=$ 114.6

Prestress force at transfer

Prestress force at 1	= 110.1 - 1.2	=	108.9 kN
Prestress force at 3	= 113.4 - 1.2	=	113.4 kN

Prestress force after all losses

Prestress force at 1	= 108.87 - 4.08 - 5.85 - 4.8	=	94.1 kN
Prestress force at 3	= 113.4 - 4.25 - 5.85 - 4.8	=	98.5 kN

Short- and long-term loss values

Average short-term loss for span 1-3
(both full-length and short tendons)

$$= \frac{22}{27} \times \frac{(14.4\% + 10.8\%)}{2} + \frac{5}{27} \times \frac{(16.4\% + 12.9\%)}{2} = \underline{13.0\%}$$

Average short-term loss for spans 3-5 and 5-7
(full-length tendons only)

$$= (10.8\% + 8.0\% + 9.8\%)/3 = \underline{9.5\%}$$

Average overall short-term loss $=$ <u>11.3%</u>

Average long-term loss for span 1-3
(both full-length and short tendons)

$$= \frac{22}{27} \times \frac{(25.8\% + 22.3\%)}{2} + \frac{5}{27} \times \frac{(27.7\% + 24.4\%)}{2} = \underline{24.4\%}$$

Average long-term loss for spans 3-5 and 5-7
(full-length tendons only)

$= (22.3\% + 19.6\% + 21.3\%)/3 \qquad = \quad \underline{21.1\%}$

Average long-term loss $\qquad = \quad \underline{22.8\%}$

Initial assumptions are fine.

A1.1.3 Designed serviceability un-tensioned reinforcement calculations

Un-tensioned reinforcement is required in the span if tensile stresses exceed $0.15\sqrt{f_{cu}}$ (see Table 2).

Transverse direction

No tensile stress at C, therefore no designed un-tensioned reinforcement is required at serviceability.

All stresses in the spans have been kept below $0.15\sqrt{f_{cu}}$ so no un-tensioned reinforcement is required in any of the spans.

At support B, there is a tensile stress and so designed un-tensioned reinforcement is required.

steel required $\quad A_s = \dfrac{F_t}{0.625 f_y}$

tensile force $\quad F_t = -\dfrac{f_{ct}(h-x)b}{2}$

$$h-x = \frac{f_{ct}}{-f_{ct} + f_{cc}} \times h$$

At B, worst stress situation is when $f_{ct} = -1.727$ N/mm²

$f_{cc} = 5.165$ N/mm² $\qquad h = 225$ mm $\qquad b = 7000$mm

$h-x = \dfrac{1.727}{6.892} \times 225 \qquad = \underline{56.38\text{mm}}$

$F_t = \dfrac{1.727 \times 56.38 \times 7000}{2} \qquad = \underline{340.79 \times 10^3 \text{ N}}$

$A_s = \dfrac{340.79 \times 10^3}{0.625 \times 460} \qquad = \underline{1185 \text{ mm}^2}$

Use 6T16 $\qquad = 1210\text{mm}^2$

Longitudinal direction

Tensile stresses at supports 3 and 5.

Support 3		Support 5	
f_{ct}	$= -1.503$ N/mm^2	f_{ct}	$= -1.121$ N/mm^2
f_{cc}	$= 5.850$ "	f_{cc}	$= 4.663$ "
h-x	= 45.99 mm	h-x	= 43.61 mm
F_t	= 241.94 kN	F_t	= 171.09 kN
A_s	= $\underline{842 \text{ mm}^2}$	A_s	= $\underline{595 \text{ mm}^2}$
Use 5T16	= 1005mm^2	Use 3T16	= 603mm^2

A1.2 Ultimate Limit State

The equivalent frame analysis at Ultimate Limit State may be carried out in accordance with Clause 3.7 of BS8110, Part 1, using the simplification of load arrangements given in Clause 3.5.2.3.

Section analysis may be carried out in accordance with Clause 4.3.7 of BS8110, Part 1.

From BS8110, Part 1, Equations 52 and 53[4].

$$x = 2.47 \times d \times \frac{f_{pu}A_{ps}}{f_{cu}bd} \times \frac{f_{pb}}{f_{pu}}$$

where: f_{pu} = characteristic strength of prestressing tendon

$$f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1-1.7\frac{f_{pu}A_{ps}}{f_{cu}bd}\right)$$

where: f_{pe} = design effective prestress in the tendons after all loses.

The value of l is taken as the full length of the tendons. This may be considered to be conservative (see Section 6.10.3).

First calculate f_{pe} and x, then calculate M_u and compare with M_A (the applied moment).

$$f_{pu} = \frac{186 \times 10^3}{100} = 1860 \text{ N/mm}^2$$

$$f_{cu} = 40 \text{ N/mm}^2$$

$$f_{pe} = \frac{P}{A_{ps}}$$

A1.2.1 Transverse direction

Applied Moments

Figure A14 shows the moment envelope for the factored dead and live loads on both spans with 20% redistribution.

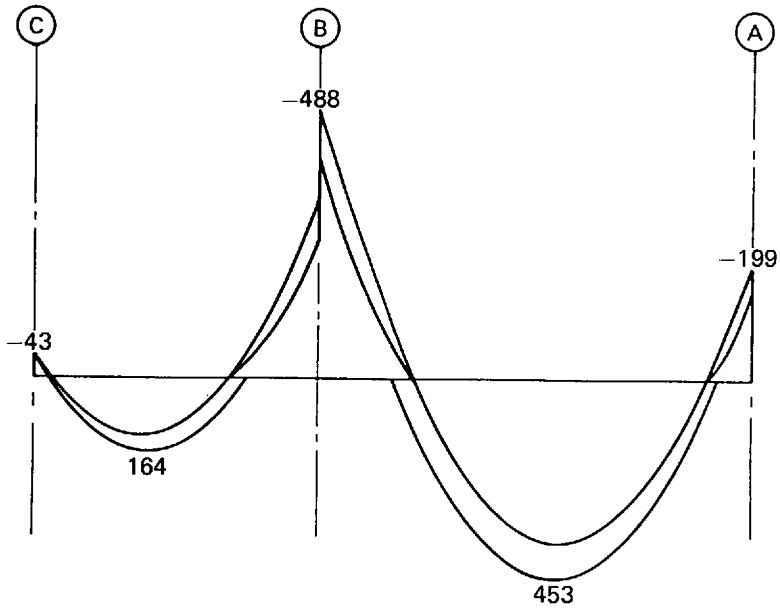


Figure A14: Applied moment envelope

Moments of Resistance

The simplified method given in Cl 4.3.7.2 of BS8110, Part 1 is used to calculate M_u at each critical section.

l = 11500 mm

	A_{ps} (mm^2)	f_{pe} (N/mm^2)	f_{pb} (N/mm^2)	x (mm)	d (mm)	d_n (mm)	M_u (kNm)
C	1100	1041.6	1102.5	10.70	112.5	4.81	-130.6
CB	1100	1041.6	1150.9	11.17	192.0	5.03	236.7
B	2600	1041.6	1130.9	25.94	176.0	11.67	-483.2
BA	2600	1041.6	1140.6	26.16	192.0	11.77	534.5
A	2600	1041.6	1092.2	25.05	112.5	11.27	-287.5

Table A11: Ultimate capacity due to prestressing tendons over the full panel width (7.0 m).

Having calculated the ultimate moment capacities above, a comparison with the total applied moment at supports and midspans must be made to find out whether further un-tensioned reinforcement is required.

The total applied moment is the sum of the moment from the Ultimate Limit State and the secondary moment. Secondary moments are discussed in more detail in Section 6.9, and methods of calculation are shown in Appendix D. The values given in Table A12 are for the full panel width (7m).

M_s	C	B	B	A
secondary moment (kNm)	28.7	10.8	40.7	83.7

Table A12: Secondary moments for the transverse direction using method B

The values shown in the above table have been calculated from method B given in Appendix D.

	M_s (kNm)	M_a (kNm)	M (kNm)	M_u (kNm)
C	28.7	-43.2	-14.6	-130.6
CB	19.7	164.3	184.0	236.7
B	10.8	-308.7	-297.8	-483.2
B	40.7	-488.6	-448.0	-483.2
BA	62.2	446.7	508.9	534.5
A	83.7	-199.1	-115.4	-287.5

Table A13: Comparison of applied transverse moments and of resistance at Ultimate Limit State

No un-tensioned reinforcement is required.

A1.2.2 Longitudinal direction

Applied Moments

Figure A15 shows the moment envelope for the factored dead and live loads on both spans with 20% redistribution.

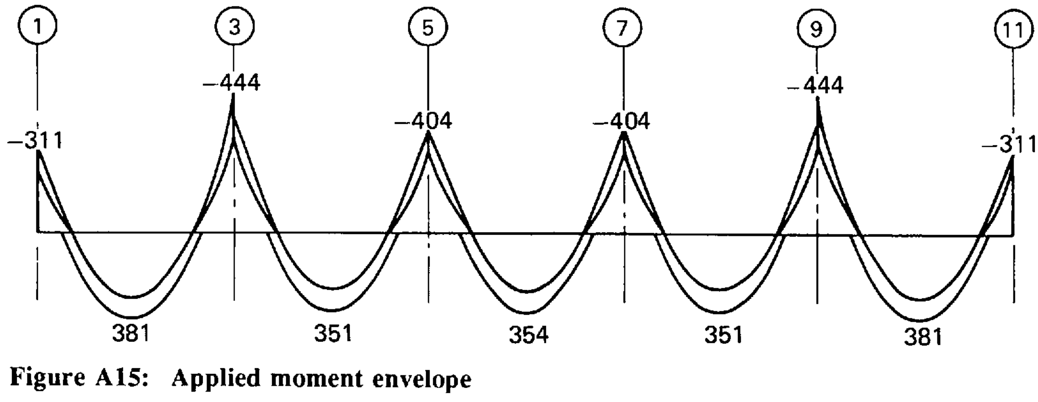


Figure A15: Applied moment envelope

l = 35000 mm

	A_{ps} (mm²)	f_{pe} (N/mm²)	f_{pb} (N/mm²)	x (mm)	d (mm)	d_n (mm)	M_u (kNm)
1	2700	1041.6	1056.7	30.64	112.5	13.79	281.6
1-3	2700	1041.6	1072.6	31.10	192.0	14.00	515.5
3	2700	1041.6	1066.2	30.91	160.0	13.91	420.5
3-5	2200	1041.6	1074.0	25.37	192.0	11.42	426.7
5	2200	1041.6	1067.6	25.22	160.0	11.35	349.1
5-7	2200	1041.6	1074.0	25.37	192.0	11.42	426.7

Table A14: Ultimate capacity due to prestressing tendons over the full panel width (6.72m).

Secondary moments and the applied moments are calculated as before.

M_s	1	3	3	5	5
secondary moment (kNm)	134.9	55.0	58.0	69.2	66.6

Table A15: Secondary moments for the longitudinal direction using method B.

	M_s (kNm)	M_a (kNm)	M (kNm)	M_u (kNm)
1	134.9	-311.5	-176.6	-281.6
1-3	94.9	381.1	476.0	515.5
3	55.0	-444.3	-389.3	-420.6
3	58.0	-413.9	-355.9	-420.6
3-5	63.6	351.5	415.1	426.7
5	69.2	-401.0	-331.9	-349.1
5	66.6	-404.3	-337.7	-349.1
5-7	66.6	354.7	421.3	426.7

Table A16: Comparison of applied longitudinal moments and of resistance at Ultimate Limit State.

No un-tensioned reinforcement is required.

A1.3 *Minimum un-tensioned reinforcement requirements*

(see also A1.1.3)

Transverse direction

Supports C, B and A

reinforcement required = 0.075% × A_c (see Section 6.10.5)

$$A_c = hb = 225 \times 7000 \qquad = 1.575 \times 10^6 \text{ mm}^2$$

$$A_s = \frac{0.075}{100} \times 1.575 \times 10^6 \qquad = \underline{1181 \text{ mm}^2}$$

Use <u>6T16</u> = 1206 mm^2

Steel positioning

Along external edges 50% of the above values are used and positioned as shown in Figures A16 and A17.

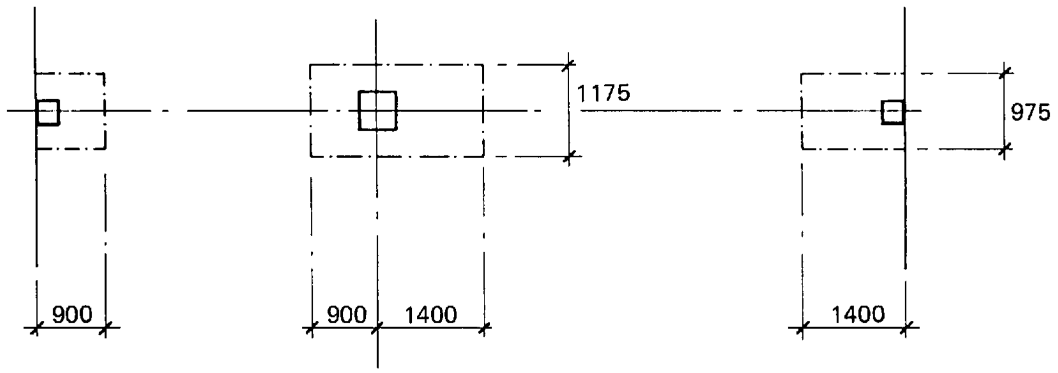


Figure A16: Transverse reinforcement positioning for internal columns

The reinforcement should extend into the span by 0.2 × span measured from the centreline of the column. The width of strip is the column breadth plus three times the slab depth.

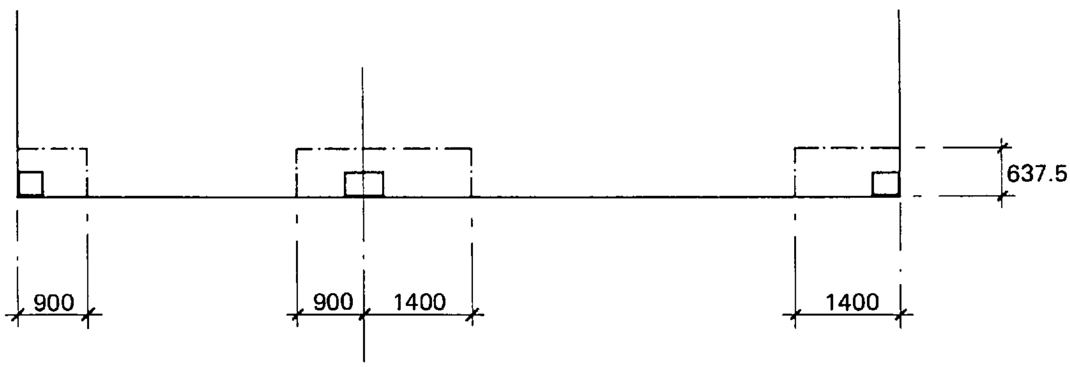


Figure A17: Reinforcement positioning for external columns

The width of strip is the column breadth plus 1.5 times the slab depth.

Longitudinal direction

Supports along grid line B.

reinforcement required $= \frac{0.075}{100} \times 225 \times 6720 \quad = \quad \underline{1134\ mm^2}$

Use $\underline{6T16}$ = 1206 mm^2

Along the external edges 50% of the above value is used.

The reinforcement is arranged in the same manner as for the transverse direction.

A1.4 Summary of prestress and un-tensioned reinforcement requirements

A1.4.1 Prestress summary

Prestress tendons are unbonded 12.9m diameter superstrand with a jacking force of 0.7 f_{pu} = 130.2 kN/tendon.

Transverse direction

Span	Tendons per panel	Prestress force
CB	11	163.7 kN
BA	26	386.9 kN

Note: Prestress force given per metre width of slab, for the service condition.

Table A17: Summary of prestress requirements in the transverse direction.

Longitudinal direction

Span	Tendons per panel	Prestress force
1-3	27	489.1 kN
3-5	22	398.5 kN
5-7	22	398.5 kN

Note: Prestress force given per metre width of slab, for the service condition.

Table A18: Summary of prestress requirements in the longitudinal direction.

A1.4.2 Un-tensioned reinforcement summary

Transverse direction

	C	CB	B	BA	A
top	6T16 (min.)	/	6T16 (min.)	/	6T16 (min.)
bottom	No bottom un-tensioned reinforcement required				

Table A19: Summary of un-tensioned reinforcement required in the transverse direction.

Longitudinal direction

	1	1-3	3	3-5	5	5-7
top	6T16 (min.)	/	6T16 (min.)	/	6T16 (min.)	/
bottom	No bottom un-tensioned reinforcement required					

Table A20: Summary of un-tensioned reinforcement required in the longitudinal direction.

A1.5 Punching Shear Check

Punching shear checks should be carried out at this stage for both internal and external columns (see Section 6.11.2.). Only an internal column check is carried out here at column B5.

70% banding of tendons in each direction is assumed (see Section 2.4).

The punching shear resistance for sections cracked in flexure is given in Section 6.11.2 as:

$$V_{cr} = v_c b_v d + M_o \frac{V}{M}$$

Unbonded tendons do not contribute to v_c. The top reinforcement provided at column B5 is given in Tables A19 and A20, and the cover to reinforcement is shown in Figure A2.

Transverse direction

Panel width	=	7m
Effective depth, $d_{tr} = 225 - 25 - 16 - 8$	=	176mm
Length of critical perimeter perpendicular to transverse direction, $b_{tr} = 500 + 3 \times 176$	=	1028mm
Area reinforcement provided (6T16s)	=	$1206mm^2$
Width over which reinforcement provided, $= 500 + 3 \times 225$	=	1175mm
Area of reinforcement provided in width, $b_{tr} = 1206 \times 1028/1175$	=	$1055mm^2$
$100\ A_s/b_{tr}d_{tr} = 100 \times 1055/(176 \times 1028)$	=	0.58%
From equation in Table 3.9 of BS8110 Pt 1 $v_{c(tr)} = (0.79/1.25) \times (0.58)^{1/3} \times (400/176)^{1/4} \times (40/25)^{1/3}$	=	$0.76N/mm^2$

Effects of Prestress. The calculation of 'e' for the purposes of the shear check is taken at the critical perimeter. For this example, the critical perimeter corresponds closely to the inflexion point. From Figure A3, the value of 'e' in the transverse direction is:

$e_{tr} = -225/2 + 49 + (25\ \text{say})$	=	-38.5mm

For Span AB		
Prestress force/tendon after all losses (see Figures A8 and A9) $= (102.7 \times 11 + 99.8 \times 15)/26$	=	101 kN
Number of tendons across panel width	=	26
Prestress force across panel width, P	=	2626 kN
Prestress force across critical perimeter, P^* $= 0.7 \times 2626$ (assuming 70% banding, see above)	=	1839 kN
Area of concrete across panel width (7m), A_c	=	$1.575\ m^2$
$Z_t^* = 1028 \times (0.225)^2/6000$	=	$0.00867m^3$
$M_o = 0.8 \times 2626 \times 0.00867/1.575 + 0.8 \times 1839 \times 38.5/1000$	=	68.2 kNm

Applied Moments and Shear Forces (see Table A13 and Appendix D)

M	=	448 kNm
$V = ((448 - 115) + (1.4 \times 8.6 + 1.6 \times 4)/7$	=	499.4 kNm
$V_{cr(AB)} = 0.76 \times 1028 \times 176/1000 + 68.2 \times 499.4/448$	=	214 kN

Similarly for Span BC

$V_{cr(BC)} = 0.76 \times 1028 \times 176/1000 + 29.3 \times 353.4/297.8$	=	172 kN

Longitudinal direction

Panel width	=	5.75m
Effective depth, d_{lg} = 225 - 25 - 8	=	192mm
Length of critical perimeter perpendicular to longitudinal direction, b_{lg} = 500 + 3 × 192	=	1076mm
Area of reinforcement provided (6T16s)	=	$1206mm^2$
Width over which reinforcement provided	=	1175mm
Area of reinforcement provided in width b_{lg} = 1206 × 1076/1175	=	$1104mm^2$
$100\ A_s/b_{lg}d_{lg}$ = 100 × 1104/(192 × 1076)	=	0.53%
Hence $v_{c(lg)} = (0.79/1.25) \times (0.53)^{1/3} \times (400/192)^{1/4} \times (40/25)^{1/3}$	=	$0.72N/mm^2$
Effect of Prestress		
e_{lg} = -225/2 + 65 + (25 say)	=	-22.5mm
For Span 3:5		
M	=	331.9 kNm
V = ((331.9 - 355.9) + 18.44 × 6.72 × 7 × 3.5)/7	=	430.3 kN
$V_{cr(3:5)}$ = 0.76 × 1076 × 192/1000 + 41.95 × 430.3/331.9	=	211 kN
For Span 5:7		
$V_{cr(5:7)}$ = 0.76 × 1076 × 192/1000 + 41.95 × 433.7/337.7	=	211 kN
Total Shear Resistance, V_{cr} = 214 + 172 + 211 + 211	=	808 kN

Applied Shear Force

Transverse direction provides worst condition		
V = 499 + 353	=	852 kN
M = 448 - 297.8	=	150 kNm
V_{eff} = V(1 + 1.5 M/Vx) see BS8110, Equation 25 where x = b_{tr}	=	1.028
V_{eff} = 852 (1 + 1.5 x 150/(1.028 × 852))	=	1071 kN
$V_{eff} > V_{cr}$ hence shear reinforcement required.		
A_{sv} required = 1000(1071 - 743)/(0.87 × 460)	=	$819mm^2$
Using T6 single leg links		
Number required = (819 × 4)/(π × 36)	=	29
Maximum spacing permitted = 1.5d = 1.5 × 176	=	264mm

Place 12 No on a perimeter of 0.5d from the column face
Place 24 No on a perimeter of 2.25d from the column face

A2 ***Example 2: One-way spanning floor with bonded and unbonded tendons***

The design philosophy used in this example is similar to that used in the first. Since it is a one-way system a preliminary shear check is considered unnecessary.

The floor plan and subframe for the structure are shown in Figure A18. It can be seen that the structure is a one-way system, with a ribbed slab spanning onto band beams. The band beams run along the column lines in the longitudinal direction. Unbonded tendons are chosen for the ribbed slabs, and bonded tendons for the band beams.

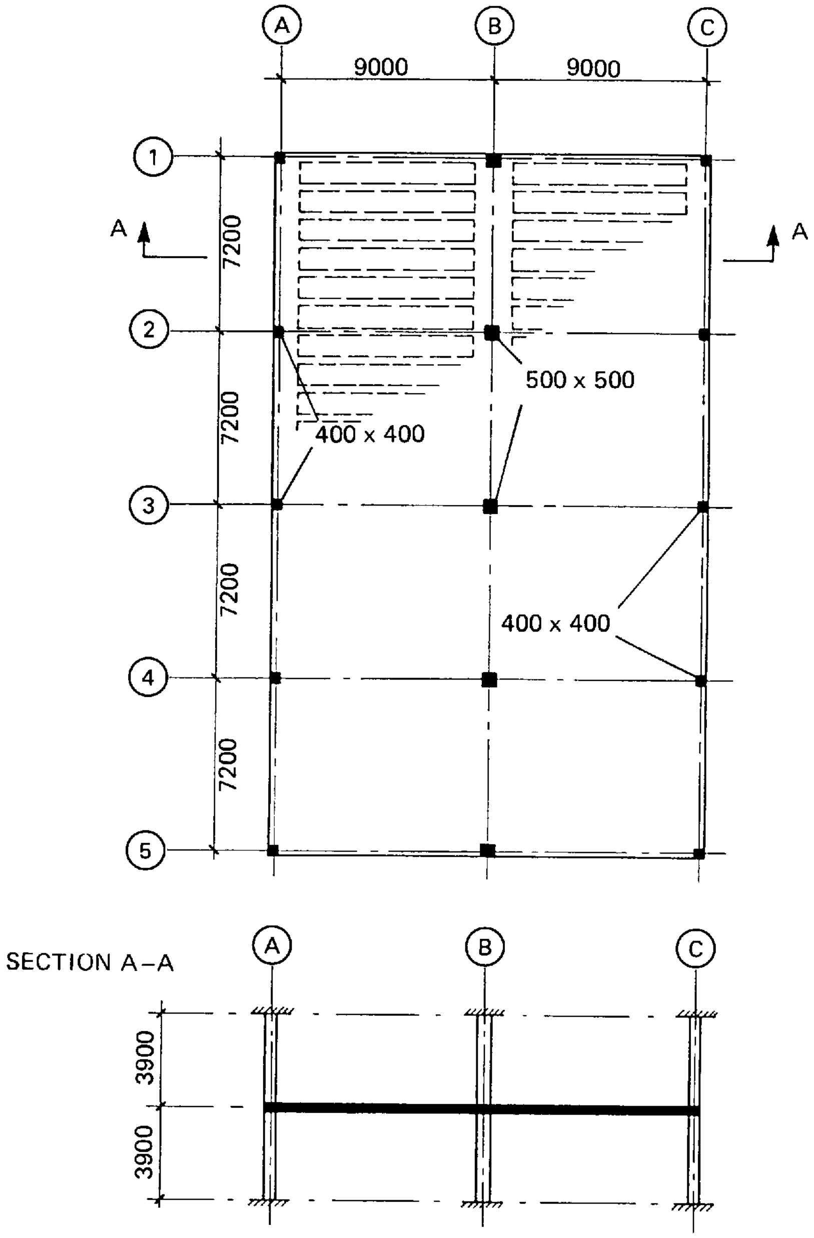


Figure A18: Floor plan and subframe for Example 2

Properties

Concrete and bonded reinforcement as for the first example.

Prestressing steel:
15.7mm diameter superstrand is used for both bonded and unbonded tendons, with a high-density polythene or polypropylene sheath surrounding the unbonded tendons, as detailed in Section 4.2.2.

P_k	$= 265.5$ kN	(characteristic force of tendon)
A_{ps}	$= 150$ mm^2	(area of tendon)
f_{pu}	$= 1770$ N/mm^2	(characteristic strength of prestressing steel)
E_{ps}	$= 195$ kN/mm^2	(elastic modulus)

Loading

Imposed loading for a typical office building:

finishes:	1.0 kN/m^2
live load:	4.0 kN/m^2
total imposed load	5.0 kN/m^2

In this example the loading from finishes will be considered as a live load. This will take account of moveable partitions.

Total live load = 5.0 kN/m^2

Using the values of span/depth ratio given in Table 1, the section dimensions are as shown in Figure A19. For analysis purposes, the ribbed section will be treated as one large T-section spanning in the transverse direction as shown in Figure A20.

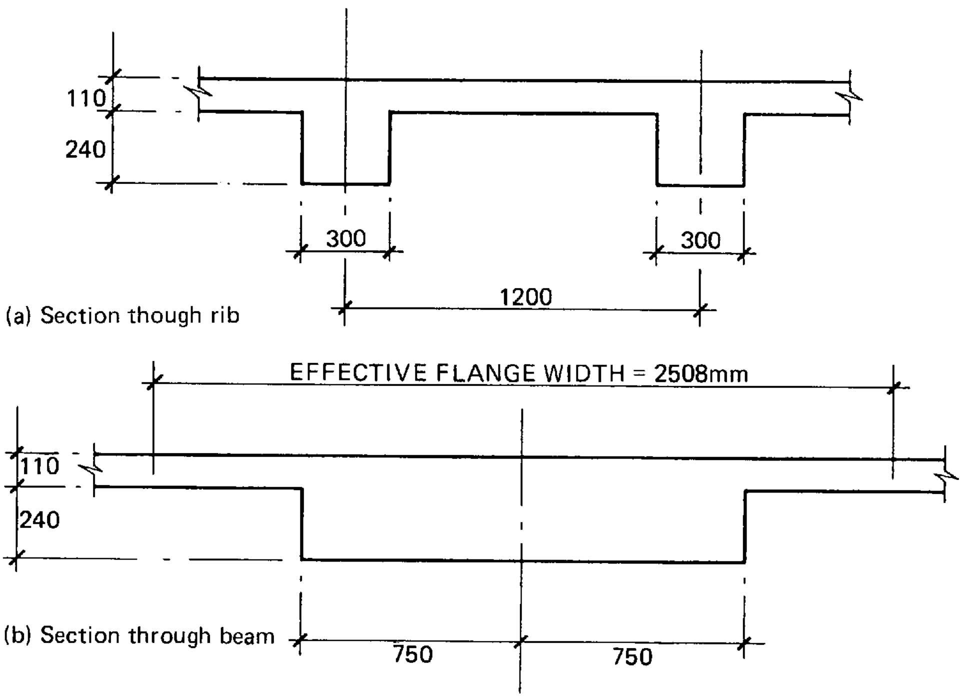


Figure A19: Section details

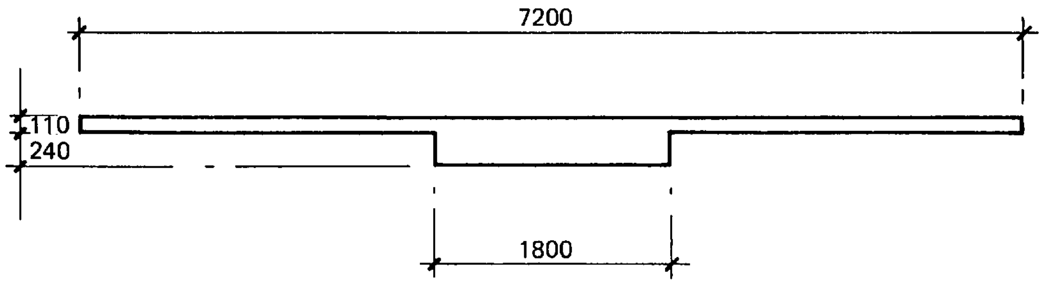


Figure A20: T-section used for analyses

Self-weight:

- transverse direction

  self-weight of T-section

  $$= (0.11 \times 7.2 \times 24) + (0.24 \times 1.8 \times 24) \quad = \quad 29.38 \text{ kN/m}$$

  Although the T-section becomes a rectangular section when it meets the band beam, for this example the T-section will be considered to run the full distance between the supports. An additional load will be applied to the T-section at the support ends, to represent this additional self-weight.

  additional weight of concrete at supports due to band beam

  $$= (0.24 \times 5.4 \times 24) \quad = \quad 31.10 \text{ kN/m}$$

- longitudinal direction

  self-weight of band beam

  $$= (0.35 \times 1.5 \times 24) \quad = \quad 12.6 \text{ kN/m}$$

Balanced load

- transverse direction

For the transverse direction take a balanced load of 1.5 times the self-weight.

balanced load = 1.5×29.38 = $\underline{44.07 \text{ kN/m}}$

- longitudinal direction

For the longitudinal direction take a balanced load equal to the self -weight.

Tendon profiles

Cover requirements as for the first example.

At this point the practical arrangement of the tendons and un-tensioned reinforcement must be considered, especially as links will be required in both the ribs and the band beams. (See Figure A18). Steel mesh will be placed in the top of the ribbed slab as a provision against

cracking, and to reinforce the concrete for a loading situation between ribs. Continuity between slab and beam will be maintained by straight bars which pass from the beam into the slab, and which are then tied to the mesh. It should be noted that the cover to tendons in ducts must not be less than 50 mm.

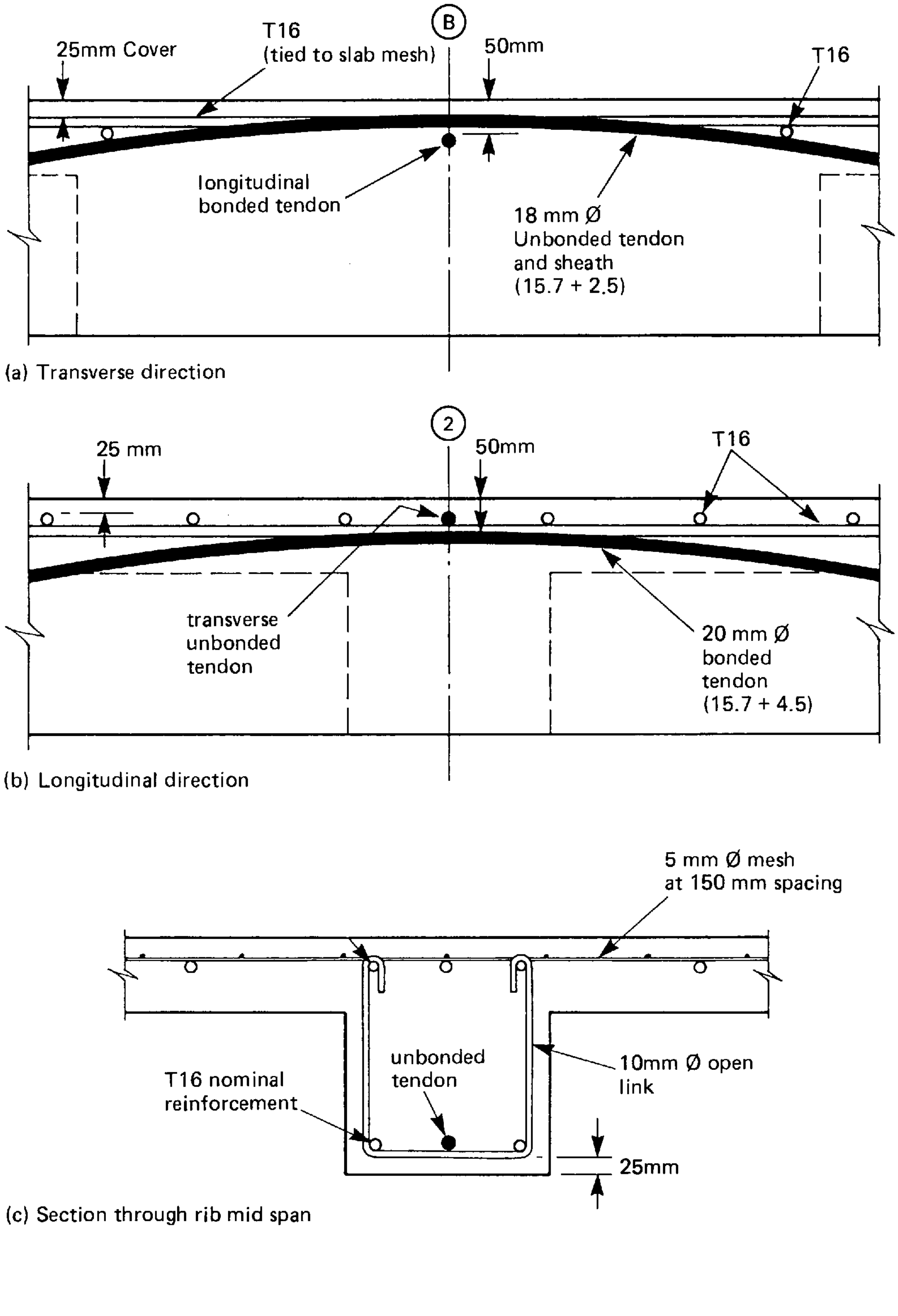


Figure A21: Tendon and reinforcement positions

A2.1 *Serviceability Limit State*

A2.1.1 Transverse direction

The tendon profile is calculated as before and the resulting profile is shown in Figure A22.

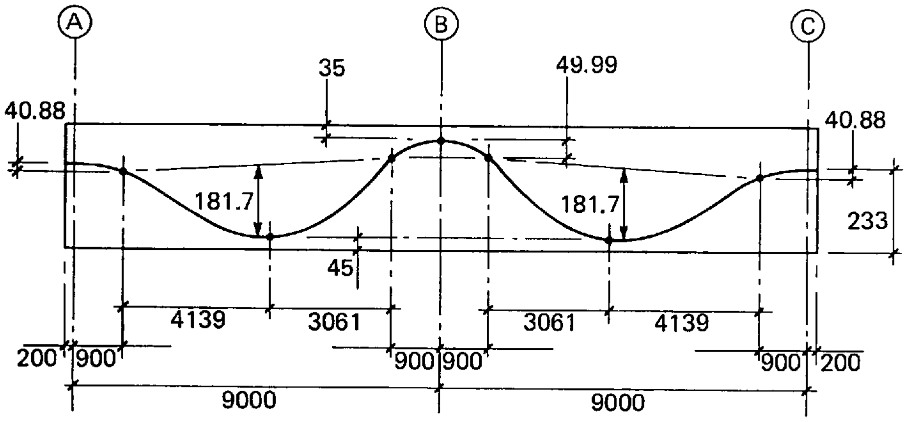


Figure A22: Transverse tendon profile

Calculation of maximum drape

From Figure A22 and Appendix C

$y = kx(s-x)$

$k = 1.40 \times 10^{-5}$ and $s = 7200$ mm

therefore, when $x = 3600$ mm, $y = 181.70$ mm

Losses are assumed to be 10% and 20% of the jacking load as before.

Initial prestress

The jacking force is taken as 70% of the characteristic strength.

Stressing of the tendons will take place along gridline A only for the transverse direction.

Jacking force in tendon $= 0.7 \times 265.5$	=	185.85 kN
Prestress force in tendon at transfer, P_{av}	=	167.27 kN
Prestress force in tendon at service, P_{av}	=	148.68 kN

Calculate the prestress force required in each span using the balanced load previously chosen and the drapes shown in Figure A23.

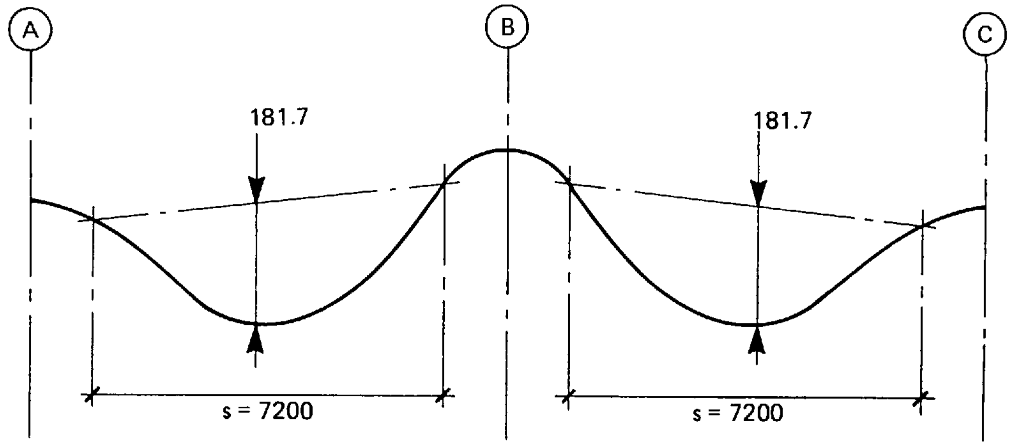


Figure A23: Drapes for load balancing

Force required is the same in each span

$$P_{reqd} = \frac{44.07 \times 7200^2}{8 \times 181.7 \times 1000} = 1571.68 \text{ kN}$$

Therefore,

$$\text{number of tendons required} = \frac{1571.68}{148.68} = 10.57$$

As there are six ribs in the section, two tendons are chosen for each rib giving a total of 12 tendons.

$$P_{av} \text{ (at transfer)} = 12 \times 167.27 = 2007.2 \text{ kN}$$

$$P_{av} \text{ (after all losses)} = 12 \times 148.68 = 1784.16 \text{ kN}$$

Equivalent loads are now calculated as for the first example.

$$w = \frac{8an\ P_{av}}{s^2}$$

where w is the equivalent load (kN)

n is the number of tendons

a is the maximum drape of tendons for zone considered (mm)

s is the distance between tendon inflexion points (mm)

A B C

Equivalent loads at transfer (n = 12)					
n × P_{av}(kN)	2007.2	2007.2	2007.2	2007.2	2007.2
a (mm)	40.9	181.7	50.0	181.7	40.9
s (mm)	1800	7200	1800	7200	1800
w (kN/m)	202.6	-56.3	247.8	-56.3	202.6
Equivalent loads after all losses (n = 12)					
n × P_{av}(kN)	1784.2	1784.2	1784.2	1784.2	1784.2
a (mm)	40.9	181.7	50.0	181.7	50.0
a (mm)	1800	7200	1800	7200	1800
w (kN/m)	180.1	-50.4	220.2	-50.2	180.1

Table A21: Calculations of equivalent loads due to transverse tendons, at transfer and after all losses, for the full slab width.

Using an appropriate computer program, the bending moments and shear forces for each case can be obtained and then the stresses calculated. As for the transverse direction in example 1 patterned loading for the SLS is used.

It should be noted that, as the section being considered is not rectangular, z_t and z_b are not equal and I is not $bd^3/12$.

By calculation it can be shown that $I = 1.14 \times 10^{-2}$ m^4 and that the centroid of the section is 233mm above the soffit.

Therefore: $z_t = 97.4 \times 10^6$ mm^3
$z_b = 48.9 \times 10^6$ mm^3

Summary of results

A B C

Equivalent loads at transfer (kN/m)	202.6	-56.3	247.8	-56.3	202.6
Equivalent loads after all losses (kN/m)	180.1	-50.0	220.2	-50.0	180.1

Table A22: Summary of equivalent loads from transverse tendons for the full slab width.

Calculation of stresses

The stresses can now be calculated. (Owing to symmetry about gridline B, the stresses at A and C will be the same, as will those in spans AB and BC.)

Area of section $= (7.2 \times 0.11) + (0.24 \times 1.8) \quad = \quad 1.224\ m^2$

Stresses at transfer:
The maximum allowable tensile and compressive stresses for one-way spanning structures at transfer are the same for both span and support locations (BS8110, Part 1: Clause 4.3.5)

max. compressive stress $= 0.5\ f_{ci} \quad = 0.5 \times 25 \quad = \quad 12.5 N/mm^2$
max. tensile stress $= 0.36\sqrt{f_{ci}} \quad = 0.36\sqrt{25} \quad = \quad 1.8\ N/mm^2$

Zone	Location	P	ΣM	f_t		f_b	
				Max	Min	Max	Min
		kN	kNm	N/mm^2	N/mm^2	N/mm^2	N/mm^2
A (support)	0.1L (A-B)	2007.2	56.3	2.22	-	-	0.49
AB (span)	0.5L (A-B)	2007.2	-92.6	-	0.69	3.53	-
	0.8L (A-B)	2007.2	24.7	1.89	-	-	1.13
B (support)	0.9L (A-B)	2007.2	107.4	2.74	-	-	-0.56
Allowable stresses - Support region (un-tensioned reinforcement top only)				12.5	-1.8	12.5	-1.8
Span region (no un-tensioned reinforcement)				12.5	-1.8	12.5	-1.8

Table A23: Stresses at transfer for the transverse direction.

Stresses after all losses:
The maximum allowable compressive and tensile stresses for one-way spanning structures after all losses are shown below, using Tables 4.2 and 4.3 of BS8110, Part 1, Clause 4.3.4.2 and Clause 4.3.4.3[4].

Take the crack width limitation to be 0.1mm.

max. tensile stress $= -4.1 \times 1.025$ $=$ -4.2 N/mm^2

max. compressive stresses:

- span locations

max. compressive stress $= 0.33f_{cu}$ $=$ 13.2 N/mm^2

- support locations

max. compressive stress $= 0.4f_{cu}$ $=$ 16.0 N/mm^2

Zone	Location	P	ΣM	f_t		f_b	
				Max	Min	Max	Min
		kN	kNm	N/mm^2	N/mm^2	N/mm^2	N/mm^2
A (support)	At A	1784.2	-111.4	-	0.31	3.74	-
AB (span)	0.5L (A-B)	1784.2	-93.51	-	0.50	3.37	-
	0.4L (A-B)	1784.2	112.2	2.61	-	-	-0.84
B (support)	At B	1784.2	-265.4	-	-1.27	6.89	-
Allowable stresses - Support region (un-tensioned reinforcement top only)				16.0	-4.2	16.0	-4.2
Span region (no un-tensioned reinforcement)				13.2	-4.2	13.2	-4.2

Table A24: Stresses after all losses for the transverse direction.

Loss calculations

- Short-term losses

a) losses due to friction

$P_x = P_o \times e^{-\mu x(\alpha' + \varpi)}$

Recommended values for the values of μ and ϖ are,

$\mu = 0.06$ and $\varpi = 0.05$ rads/m

total drape $= \dfrac{40.88 + 49.99}{2} + 181.7$ $=$ 227.14 mm

deviated angle per metre, α', is the same for both spans

α'	=	$\frac{16 \times 227.14 \times 10^{-3}}{9^2}$	=	0.045 rads/m
P_A	=	jacking force	=	185.9 kN
P_B	=	$185.85 \times e^{-9 \times 0.06(0.045 + 0.05)}$	=	176.6 kN
P_C	=	$176.56 \times e^{-9 \times 0.06(0.045 + 0.05)}$	=	167.7 kN

b) losses due to wedge set

$$\delta P_w = 2p'l'$$

where: $l' = \sqrt{((\Delta \times E_{ps} \times A_{ps})/p')}$

and $p' = \frac{P_A - P_C}{L_1 + L_2} = 1.01$ kN/m

take Δ = 6mm

therefore: l' = 13.20 m

and δP_w = 26.66 kN at the stressing anchor

Forces after friction losses and wedge set: (see Figure A24)

P_A	=	159.2 kN
P_B	=	168.2 kN
P_C	=	167.7 kN

c) elastic losses

$$\delta P_{es} = \varepsilon_{es} \times E_{ps} \times A_{ps}$$

$$\varepsilon_{es} = 0.5 \times \frac{f_{co}}{E_{ci}}$$

from stress calculations, $f_{co} = 2.183$ N/mm^2

therefore:

$$\varepsilon_{es} = 5.03 \times 10^{-5}$$

and δP_{es} = 1.47 kN

As this is less than 1% of the initial force, elastic losses will be ignored.

Prestress at transfer

Prestress force at A	=	159.2 kN
Prestress force at B	=	168.2 kN
Prestress force at C	=	167.7 kN

-Long-term losses

a) Relaxation of steel

δP_r = 3.75% as for the first example

δP_{rA} = 5.97 kN
δP_{rB} = 6.31 kN
δP_{rC} = 6.29 kN

b) Shrinkage of concrete

$\delta P_{sh} = \varepsilon_{sh} \times E_{ps} \times A_{ps}$

$\varepsilon_{sh} = 300 \times 10^{-6}$

δP_{sh} = 8.78 kN

c) Creep of concrete

$\delta P_{cr} = \varepsilon_{cc} \times E_{ps} \times A_{ps}$

$$\varepsilon_{cc} = \frac{f_{co}}{E_{ci}} \times \phi$$

ϕ = 1.6

f_{co} = 2.183 N/mm^2

δP_{cr} = 4.71 kN

Prestress after all losses

Prestress force at A = 159.19 - 5.97 - 8.78 - 4.71	=	139.7 kN
Prestress force at B = 168.22 - 6.31 - 8.78 - 4.71	=	148.4 kN
Prestress force at C = 167.73 - 6.29 - 8.78 - 4.71	=	147.9 kN

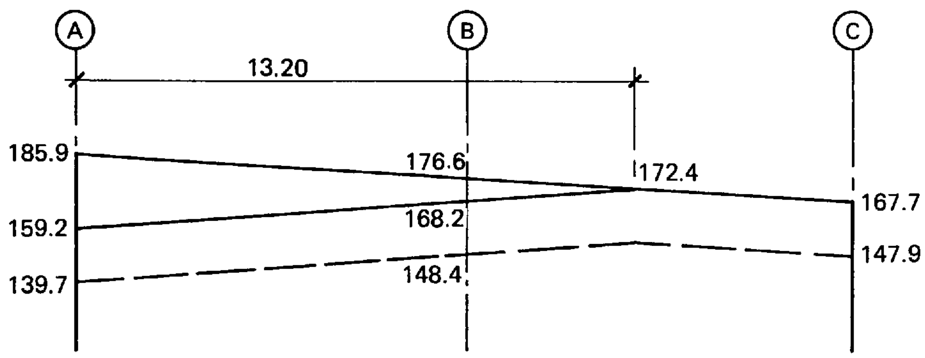


Figure A24: Force profiles

Short- and long-term losses

average short-term loss	= (14.3% + 9.5% + 9.7%)/3	=	<u>11.2%</u>
average long-term loss	= (24.8% + 20.1% + 20.4%)/3	=	<u>21.8%</u>

Although the initial values of 10% and 20% have been exceeded, the calculations will not be revised. This is because the stresses at transfer are not close to exceeding any of the allowable limits, and the number of tendons would not be affected if the losses were increased by 1-2%.

A2.1.2 Longitudinal direction

The effective width of slab was found from the transverse direction shear force diagram for full unfactored load.

$= 10.22$ m

Self-weight = Weight of slab + Weight of beam

$$= \frac{(10.22 - 1.5) \times 29.38}{7.2} + 12.6$$

$= 48.20$ kN/m

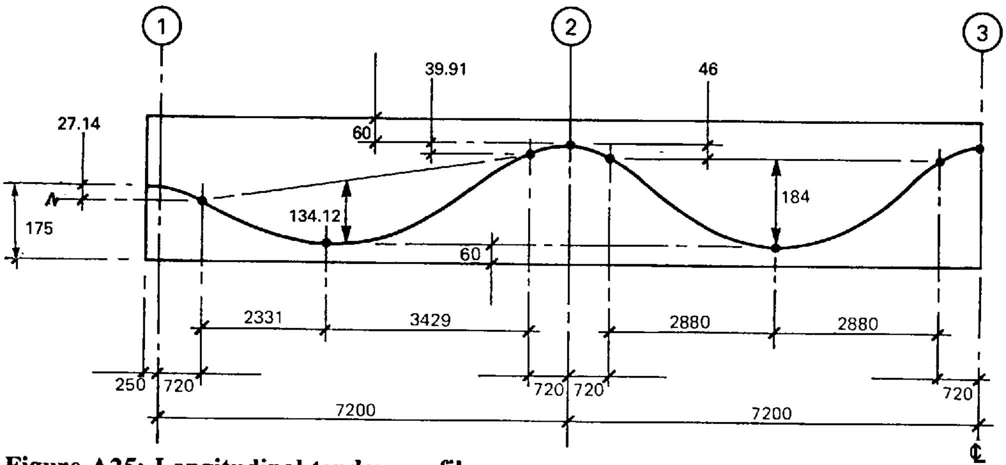


Figure A25: Longitudinal tendon profile

In this example the centroid of the tendons was taken to be the same as the centroid of the duct, as with this arrangement (20mm oval duct with 15.7mm tendons) the difference in eccentricity is negligible. (See section 6.7 of this manual.)

Calculation of maximum drape in each span.

Span 1-2
From Figure A.25 and Appendix C

$y = kx(s-x)$

$k = 1.74 \times 10^{-5}$ and $s = 5760$mm

Maximum drape assumed to be at $x = L/2 = 2880$
Maximum drape, $y = 144$ mm
A detailed calculation would show the maximum drape occurs at x = 2331 mm and is 139 mm as shown in Figure A25.

Span 2-3
Maximum drape $= 350 - 60 - 46 - 60 = 184$mm

Losses assumed to be:

15% of the jacking load at transfer
25% of the jacking load after all losses

These are higher than for the transverse direction because losses tend to be higher for bonded systems in beams.

Initial prestress

Prestress force at transfer, P_{av}	=	158.0 kN
Prestress force at service, P_{av}	=	139.4 kN

Calculate prestress force required for each span

Balanced load, w = 48.2 kN/m

Spans 1-2 and 4-5

$$P_{reqd} = \frac{48.2 \times 5760^2}{8 \times 144 \times 1000} = 1388.2 \text{ kN}$$

Spans 2-3 and 3-4

$$P_{reqd} = \frac{48.2 \times 5760^2}{8 \times 184 \times 1000} = 1086.4 \text{ kN}$$

For a bonded structure, construction is easier if the number of tendons in each span is equal. Although ten tendons would be just sufficient from this approximate calculation, the choice of 11 will ensure no overstressing in the outer spans even if the calculation of final losses exceeds 25%.

For this analysis it is considered reasonable to calculate the effective flange width for stiffness and stresses according to BS8110 Clause 3.4.1.5, i.e. web width plus 0.7 times span/5. The loading width is, of course determined by the analysis in the transverse direction.

As the beam is a T-section, z_t and z_b are not equal. By calculation it can be shown that $I = 6.79 \times 10^{-3}$ m^4 and that the centroid of the section is 196 mm above the soffit. $A = 0.6359$ m^2.

Therefore: $z_t = 44.1 \times 10^6$ mm^3
$z_b = 34.6 \times 10^6$ mm^3

	1		2			3
Equivalent loads at transfer (n = 11)						
$n \times P_{av}$(kN)	1737.6	1737.6	1737.6	1737.6	1737.6	1737.6
a (mm)	30.8	-144.0	41.2	46.00	-184.0	46.00
s (mm)	1440	5760	1440	1440	5760	1440
w (kN/m)	206.5	-60.3	276.2	308.4	-77.1	308.4
Equivalent loads after all losses (n = 11)						
$n \times P_{av}$(kN)	1533.3	1533.3	1533.3	1533.3	1533.3	1533.3
a (mm)	30.8	-144.0	41.2	46.00	-184.0	46.00
a (mm)	1440	5760	1440	1440	5760	1440
w (kN/m)	182.2	-53.2	243.7	272.1	-68.0	272.1

Table A25: Calculations of equivalent loads due to longitudinal tendons, at transfer and after all losses, for the full slab width.

Summary of equivalent loads

	1		2			3
Equivalent loads at transfer (kN/m)	206.5	-60.3	276.2	308.4	-77.1	308.4
Equivalent loads after all losses (kN/m)	182.2	-53.2	243.7	272.1	-68.0	272.1

Table A26: Summary of equivalent loads from longitudinal tendons for the beam.

Calculation of stresses

Zone	Location	P	ΣM	f_t		f_b	
				Max	Min	Max	Min
		kN	kNm	N/mm²	N/mm²	N/mm²	N/mm²
1 (support)	At 1	1737.6	27.21	3.35	-	-	1.95
1-2 (span)	0.5L (1-2)	1737.6	-15.80	-	2.37	3.19	-
	0.8L (1-2)	1737.6	17.81	3.14	-	-	2.22
2 (support)	0.9L (1-2)	1737.6	41.56	3.67	-	-	1.53
	0.1L (2-3)	1737.6	52.36	3.92	-	-	1.22
2-3 (span)	0.5L (2-3)	1737.6	-59.27	-	1.39	4.45	-
	0.8L (2-3)	1737.6	14.32	3.06	-	-	2.32
3 (support)	0.9L (2-3)	1737.6	68.82	4.29	-	-	0.75
Allowable stresses - Support region (un-tensioned reinforcement top only)				12.5	-1.8	12.5	-1.8
Span region (no un-tensioned reinforcement)				12.5	-1.8	12.5	-1.8

Table A27: Stresses at transfer for the longitudinal direction.

Allowable stress calculations as for the transverse direction.

Zone	Location	P	ΣM	f_t		f_b	
				Max	Min	Max	Min
		kN	kNm	N/mm²	N/mm²	N/mm²	N/mm²
1 (support)	At 1	1533.3	-210.3	-	-2.34	8.50	-
1-2 (span)	0.5L (1-2)	1533.3	146.0	5.72	-	-	-1.81
	0.8L (1-2)	1533.3	-24.81	-	1.85	3.13	-
2 (support)	At 2	1533.3	-286.4	-	-4.08	10.69	-
	At 2	1533.3	-237.6	-	-2.98	9.28	-
2-3 (span)	0.5L (2-3)	1533.3	101.10	4.70	-	-	-0.51
	0.5L (2-3)	1533.3	-72.89	-	0.76	4.52	-
3 (support)	At 2	1533.3	-207.6	-	-2.30	8.41	-
Allowable stresses - Support region (un-tensioned reinforcement top only)				16.0	-4.2	16.0	-4.2
Span region (no un-tensioned reinforcement)				13.2	-4.2	13.2	-4.2

Table A28: Stresses at transfer after all losses for the longitudinal direction.

Allowable stress calculations as for the transverse direction.

Loss calculations

- Short-term losses

a) Losses due to friction

$$P_x = P_o e^{-\mu x(\alpha' + \varpi)}$$

take recommended values of: $\mu = 0.2$
$\varpi = 0.0085$ rads/m

spans 1-2 and 4-5
total drape = 167.65mm

$$\alpha' = \frac{16 \times 180 \times 10^{-3}}{7.2^2} = 0.056 \text{ rads/m}$$

spans 2-3 and 3-4
total drape = 230mm

$$\alpha' = \frac{16 \times 230 \times 10^{-3}}{7.2^2} = 0.071 \text{ rads/m}$$

jacking force = 185.85 kN

(See Figure A26)

P_1	=	jacking force	=	185.8 kN
P_2	=	$185.85\ e^{-0.2 \times 7.2(0.056 + 0.0085)}$	=	169.4 kN
P_3	=	$169.37e^{-0.2 \times 7.2(0.0.071 + 0.0085)}$	=	151.0 kN

b) Losses due to wedge set

Take Δ = 6mm

p' = 2.37 kN/m
l' = 8.61 m

therefore δP_w = 40.81 kN

c) Elastic losses

Elastic losses are less than 1% and are ignored in the calculations.

Prestress at transfer

Prestress force at 1	=	145.0 kN
Prestress force at 2	=	162.5 kN
Prestress force at 3	=	151.0 kN

- Long-term losses

a) Relaxation of steel

3.75% as before

δP_{r1} = 5.44 kN
δP_{r2} = 6.09 kN
δP_{r3} = 5.66 kN

b) Shrinkage of concrete

δP_{sh} = 8.78 kN as before

c) Creep of concrete

$$\delta P_{cr} = \varepsilon_{cc} \times E_{ps} \times A_{ps}$$

$$\delta P_{cr} = 1.6 \times \frac{1.579 \times 195 \times 150}{21.7 \times 10^3}$$

$$\delta P_{cr} = 3.41 \text{ kN}$$

Prestress after all losses

Prestress force at 1 = 145.06 - 5.44 - 8.78 - 3.41	=	127.4 kN
Prestress force at 2 = 162.47 - 6.09 - 8.78 - 3.41	=	144.2 kN
Prestress force at 3 = 151.70 - 5.66 - 8.78 - 3.41	=	133.8 kN

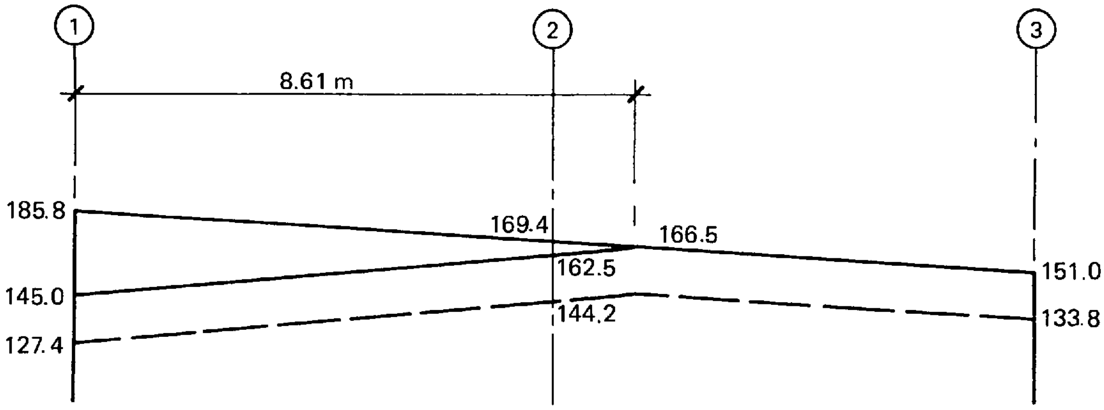


Figure A26: Force profiles.

Short- and long-term losses

average short-term loss	=	(21.9% + 12.6% + 18.8%)/3	=	<u>17.8%</u>
average long-term loss	=	(31.4% + 22.4% + 28.0%)/3	=	<u>27.3%</u>

Initial assumptions of 15% and 25% losses respectively are reasonably accurate and no further calculations are considered necessary.

A2.1.3 Serviceability un-tensioned reinforcement calculations

Transverse direction

At B

Bottom steel from tensile stresses at transfer.

f_{ct} = -0.56 N/mm^2
f_{cc} = 2.74 N/mm^2

h = 350mm and b = 1800mm

therefore:

x = 59.15mm

F_t = 29.70 kN

A_s = 103mm^2

Top steel from tensile stresses after all losses.

f_{ct} = -1.267 N/mm^2
f_{cc} = 6.885 N/mm^2

b = 7200 mm

x = 54.40mm
F_t = 248.12 kN
A_s = 863mm^2

Span AB

Bottom steel from tensile stress at service.

f_{ct}	=	$-0.835\ N/mm^2$
f_{cc}	=	$2.609\ N/mm^2$
b	=	1800 mm
x	=	84.86 mm
F_t	=	63.77 kN
A_s	=	$222\ mm^2$

As the ribs will contain links, two bars will be needed in the top and bottom of each rib although the actual amount of reinforcement was calculated to be much less.

Longitudinal direction

No un-tensioned reinforcement is required at serviceability in this direction.

A2.2 Ultimate Limit State

A2.2.1 Transverse direction

Applied moments: The simplified method of analysis is assumed using the single load case given in BS8110 Pt.1 clause 3.5.2.3 with appropriate moment redistribution.

$$M_u = f_{pb} \times A_{ps} \times (d - d_n)$$

where:

$$d_n = 0.45x$$

$$x = 2.47 \times d \times \frac{f_{pu}A_{ps}}{f_{cu}bd} \times \frac{f_{pb}}{f_{pu}}$$

$$f_{pb} = f_{pe} + \frac{7000}{l/d}\left(1 - 1.7\frac{f_{pu}A_{ps}}{f_{cu}bd}\right)$$

f_{pu}	=	$1770\ N/mm^2$
f_{cu}	=	$40\ N/mm^2$
f_{pe}	=	$991.2\ N/mm^2$
A_{ps}	=	$1800\ mm^2$
l	=	18000 mm

at supports,	b	=	1800 mm	(total rib width)
in spans,	b	=	7200 mm	(flange width)

	d (mm)	f_{pb} (N/mm²)	x (mm)	d_n (mm)	M_u (kNm)
A	233.0	1052.56	65.00	29.25	386.3
AB	305.0	1102.50	17.02	7.66	590.1
B	315.0	1084.45	66.96	30.13	556.1

Table A29: Ultimate capacity due to prestressing tendons over full panel width (7.2m).

Calculate the secondary moments using method B to obtain the applied moment at Ultimate Limit State.

	A	B
secondary moment (kNm)	115.7	157.5

Table A30: Secondary moments for the transverse direction.

Compare the moment of resistance of the tendons with the total applied moment to see whether un-tensioned reinforcement is required.

	Moment from ULS (kNm)	Secondary moment (kNm)	Applied moment (kNm)	Redistributed M (kNm)	M_u (kNm)
A	-315.2	115.7	-199.5	-159.6	-386.0
AB	439.6	136.6	576.2	691.4	590.1
B	-858.0	157.5	-700.4	-560.4	-556.1

Table A31: Comparison of applied and ultimate moments.

Reinforcement is required in span and at B as M_u has been exceeded.

A2.2.2 Longitudinal direction

f_{pu}	= 1770 N/mm²
f_{cu}	= 40 N/mm²
f_{pe}	= 929.27 N/mm²
A_{ps}	= 1650 mm²
l	= 28800 mm
b	= 1500 mm at supports
	= 2508 mm at midspan
f_{pe}/f_{pu}	= 0.525

	d (mm)	$\frac{f_{pu}A_{ps}}{f_{cu}bd}$	f_{pb} (N/mm²)	x (mm)	d_n (mm)	M_u (kNm)
1	196	0.25	1324.30	92.10	41.50	337.6
1-2	290	0.10	1539.90	63.80	287.00	663.9
2	290	0.17	1493.70	92.80	41.80	611.6

Table A32: Ultimate moment capacity due to prestressing tendons over the full panel width (1.5 m).

The value of M_u for support 3 is the same as for support 2, and the value of M_u for span 2-3 is the same as that for span 1-2. This is because the values of d and b do not vary at these points.

	1	2	2	3
secondary moment (kNm)	118.8	47.6	62.6	73.8

Table A33: Secondary moments for the longitudinal direction

Compare the applied moment to the moment of resistance to check whether un-tensioned reinforcement is required.

	Moment from ULS (kNm)	Secondary moment (kNm)	Applied moment (kNm)	M_u (kNm)
1	118.8	-503.8	-385.0	-337.6
1-2	83.2	409.5	492.7	663.9
2	47.6	-716.2	-668.6	-611.6
2	62.6	-665.5	-602.9	-611.6
2-3	68.2	381.6	449.8	663.9
3	73.8	-634.3	-560.5	-611.6

Table A34: Comparison of applied and ultimate moments.

Un-tensioned reinforcement required at supports 1 and 2, and therefore 4 and 5.

No moment redistribution has been applied.

A2.2.3 Ultimate un-tensioned reinforcement calculations

Transverse direction.

Span AB

20% redistrbution

$\beta_b = \frac{666.2}{576.21} = 1.16$

$k' = 0.402(1.16 - 0.4) - 0.18(1.16 - 0.4)^2 = 0.201$

$k = \frac{M_{AB}}{bd^2f_{cu}} = \frac{666.2 \times 10^6}{7200 \times 290^2 \times 40} = 0.028$

As k < k', compression reinforcement not required.

$z = d(0.5 + \sqrt{(0.25 - 0.028/0.9)}) \leq 0.95d$

$d = 290$ mm

$z = 290 \times 0.97$ mm

Therefore

$z = 0.95d = 275.5$ mm

$A_s = \frac{M_{AB} - M_u}{0.87f_yz}$

$= \frac{(662.2 - 590.1) \times 10^6}{0.87 \times 460 \times 275.5}$

$A_s = 681.1\ mm^2$

Therefore

$A_s = \frac{681.1}{6} = 113.5\ mm^2$

At B

20% redistribution

$\beta_b = \frac{560.38}{700.91} = 0.80$

$k' = 0.402(0.80 - 0.4) - 0.18(0.80 - 0.4)^2$

$= 0.132$

$k = \frac{M_B}{bd^2f_{cu}} = \frac{560.38 \times 10^6}{1800 \times 290^2 \times 40} = 0.093$

As $k < k'$, compression reinforcement not required.

$$z = d(0.5 + \sqrt{(0.25 - 0.093/0.9)}) \leq 0.95d$$
$$d = 290 \text{ mm}$$
$$z = 290 \times 0.88 \text{ mm}$$
$$= 256.1 \text{ mm}$$

$$A_s = \frac{M_B - M_u}{0.87 f_y z}$$

$$= \frac{(560.38 - 556.06) \times 10^6}{0.87 \times 460 \times 256.1}$$

$$A_s = 42.1 \text{ mm}^2$$

Therefore

$$A_s = \frac{42.1}{6} = 7.02 \text{ mm}^2 \text{ per rib}$$

Longitudinal direction.

Support 1

$$k' = 0.156$$

$$k = \frac{M_1}{bd^2 f_{cu}} = \frac{385.0 \times 10^6}{1500 \times 196^2 \times 40} = 0.167$$

As $k > k'$, compression reinforcement is required.

$$z = d(0.5 + \sqrt{(0.25 - k'/0.9)})$$
$$d = 196 \text{ mm and } d' = 35 \text{ mm}$$
$$z = 152.3 \text{ mm}$$

$$A_s' = \frac{(k - k') f_{cu} bd^2}{0.87 f_y (d - d')}$$

$$A_s' = \frac{(0.167 - 0.156) \times 40 \times 1500 \times 196^2}{0.87 \times 460 \times (196 - 35)}$$

$$A_s' = 394 \text{ mm}^2$$

$$A_s = \frac{(k' f_{cu} bd^2 - M_u)}{0.87 f_y z} + A_s'$$

$$A_s = \frac{(0.156 \times 40 \times 1500 \times 196^2) - 337.6 \times 10^6}{0.87 \times 460 \times 152.3} + 394$$

$$A_s = 755 \text{ mm}^2$$

Support 2

$$k' = 0.156$$

$$k = \frac{M_2}{bd^2f_{cu}} = \frac{668.6 \times 10^6}{1500 \times 290^2 \times 40} = 0.133$$

As k < k', compression reinforcement not required.

$$z = d(0.5 + \sqrt{(0.25 - k/0.9)}) \leq 0.95d$$

$$d = 290 \text{ mm}$$

$$z = 237.7 \text{ mm}$$

$$0.95d = 275.5 \text{ mm}$$

$$A_s = \frac{M_2 - M_u}{0.87f_yz}$$

$$A_s = \frac{(668.6 - 611.6) \times 10^6}{0.87 \times 460 \times 237.7} = \underline{599.1 \text{ mm}^2}$$

A2.3 *Minimum un-tensioned reinforcement requirements*

Transverse direction (see section 6.10.6)

For unbonded tendons, minimum reinforcement is required in accordance with BS8110, Part 1, Table 3.27, Figures 3.24 and 3.25[(4)].

From Table 3.27

At supports: flange in tension

$$A_s = 0.26\%b_wh$$

$$= 0.26\% \times 1800 \times 350$$

$$= 1638 \text{ mm}^2$$

$$\text{Therefore, } A_s = \frac{1638}{6} = \underline{273 \text{ mm}^2 \text{ per rib}}$$

In spans: Flange in compression

$$\frac{b_w}{b} = \frac{1800}{7200} = 0.25$$

$$\text{Therefore, } A_s = 0.18\% \times b_w \times h$$

$$= 0.18\% \times 1800 \times 350$$

$$= 1134 \text{ mm}^2$$

$$\text{Therefore, } A_s = \frac{1134}{6} = \underline{189 \text{ mm}^2 \text{ per rib}}$$

Longitudinal direction

For bonded tendons, no minimum un-tensioned reinforcement is required see Section 6.10.6.

A2.4 Summary of prestress and un-tensioned reinforcement requirements

A2.4.1 Prestress Summary

Jacking force 185.5 kN per tendon.

Transverse direction
Two tendons required in each rib

Longitudinal direction
11 tendons required throughout band beam

A2.4.2 Un-tensioned Reinforcement Summary

Position	Reinforcement required				Reinforcement required	
	Area for SLS (mm^2)	Minimum area (mm^2)	Area for ULS (mm^2)	Practical consideration for links	Bars required top	Bars required bottom
A	-	273	-	2T16 (402)	2T16 (402)	2T16 (402)
AB	-	189	113.5	2T16 (402)	2T16 (402)	2T16 (402)
B	-	273	7.0	2T16 (402)	2T16 (402)	2T16 (402)

Table A35: Transverse direction. Areas and bars given per rib.

	Reinforcement required				Reinforcement provided	
Position	**Area for SLS (mm^2)**	**Minimum area (mm^2)**	**Area for ULS (mm^2)**	**Practical consideration for links**	**Bars required top**	**Bars required bottom**
1	-	-	394 bot 755 top	2T16 (402)	2T16 (804)	2T16 (402)
1-2	-	-	-	2T16 (402)	2T16 (402)	2T16 (402)
2	-	-	599.1	2T16 (402)	2T16 (603)	2T16 (402)
2-3	-	-	-	2T16 (402)	2T16 (402)	2T16 (402)
3	-	-	-	2T16 (402)	2T16 (402)	2T16 (402)

Table A36: Longitudinal direction. Areas and bars given per beam.

For calculation of anchorage bursting reinforcement, see Appendix E, example 2.

APPENDIX B: Calculation of Prestress Losses

Friction Losses in the Tendon

Friction losses can be calculated in accordance with BS8110[4]. However, it is the view of the Working Party that the following calculations are more realistic. The losses are due to the friction resulting from the change in angle of the tendon and unintentional 'wobble' in the tendon. Both effects are considered in the common formula for friction, viz.

$$P_x = P_o e^{-\mu(\alpha+\varpi x)}$$

where: P_x = force at distance x from stressing end
P_o = stressing force (at anchor)
μ = friction coefficient
α = angle change in tendon from anchor to point considered (radians)
ϖ = 'wobble' factor (radians/m)

This is equivalent to equations 58 and 59 in BS8110, Part 1, Clause 4.9[4].

The values of the friction coefficient will depend on the prestressing system chosen and, in the case of bonded systems, the state of the strand in terms of rust film[27].

In the absence of detailed information on friction coefficient and wobble factor from the prestress system supplier, it is recommended that the factors in Table B1 are used:

	Unbonded tendons	Bonded tendons
friction coefficient μ	0.06	0.20
wobble ϖ (rad/m)	0.05	0.0085

Table B1: Typical friction coefficients and wobble factors.

For slab type structures with unbonded tendons it is normally reasonable to assume a uniform angle change per unit length. This angle change can be obtained by calculating the total angle turned through over the full length of the tendon and dividing by the full tendon length. Alternatively a simple method based on the typical drape and span can be used. Figure B1 illustrates the geometry for a typical parabolic tendon with a reverse parabola at the support. The tangent to the curve at the point of inflection extends through points 'c' and 'a'.

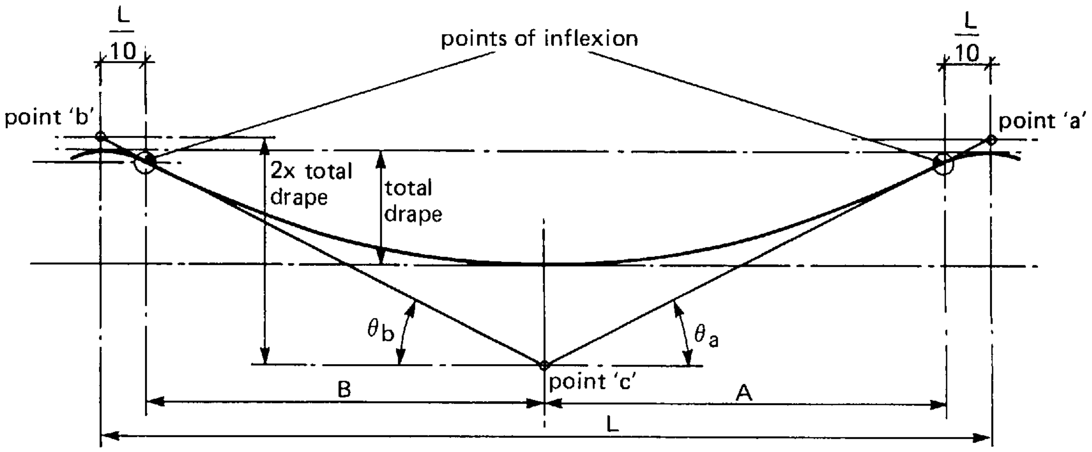


Figure B1: Typical geometry of tendon profile for internal span.

Thus the slope $\theta_a \quad = \quad \tan^{-1}((2 \times \text{total drape})/A)$

Similarly, using points 'b' and 'c', slope θ_b can be obtained.
Over the span L the total deviated angle = $2(\theta_a+\theta_b)$

The average deviated angle per unit length, α', is therefore

$$\alpha' = 2(\theta_a+\theta_b)/L$$

On the assumption that point 'c' is at the centre of the span, this may be simplified to:

$$\alpha' = \frac{16 \times \text{total drape}}{L^2}$$

In such cases, the friction formula may be written as:

$$P_x = P_o e^{-\mu x(\alpha'+\varpi)}$$

The prestress force profile after friction losses can now be drawn.

Wedge set or draw-in

Most post-tensioning systems used in buildings depend on a wedge-based system for anchoring. In order for the wedges to grip, there must be a small movement of the strand into the anchorages. This inward movement reduces the prestress and the amount of movement depends on the particular prestressing system employed; a typical value is 6 mm. The draw-in effect is as shown in Figure B2.

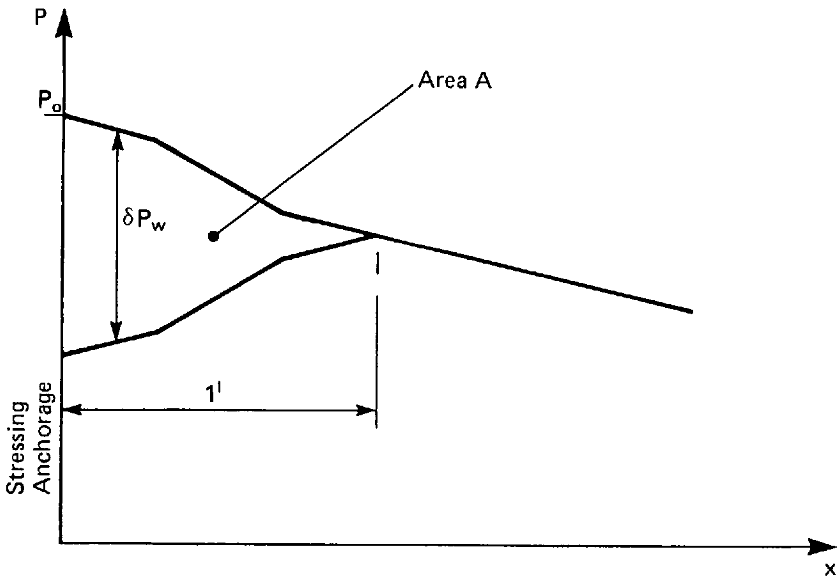


Figure B2: Loss of prestress due to wedge draw-in

The force loss is calculated as follows:

$$\text{Area } A = \Delta \times E_{ps} \times A_{ps}$$

$$= \int_0^{l'} \delta P_w \, dx$$

where:

Δ	=	wedge draw-in
δP_w	=	force loss
E_{ps}	=	modulus of elasticity of tendon
A_{ps}	=	area of tendon
l'	=	length of tendon affected by draw-in

If it can be assumed that the tendon has a uniform angle change per unit length, then the force profile is approximately linear. Consequently, if l' is less than the length of the tendons, then:

$$l' = \sqrt{((\Delta \times E_{ps} \times A_{ps})/p')}$$

where: p' = slope of the force profile

and

$$\delta P_w \text{ at anchorage} = 2 \times p' \times l'$$

The force loss, within the length l', is then given by:

$$\delta P_w = 2p'(l' - x)$$

If the wedge draw-in affects the whole length of the tendon, then:

$$\delta P_w \text{ at stressing anchorage} = (\Delta \times E_{ps} \times A_{ps})/l + p' \times l$$

$$\delta P_w \text{ at dead end} = (\Delta \times E_{ps} \times A_{ps})/l - p' \times l$$

Elastic Shortening of the Structure

As strands are tensioned, the structure will shorten elastically. In most building floors, this shortening is insignificant in terms of losses, but it may be significant in highly stressed beams.

The force loss is given by:

$$\delta P_{es} = \varepsilon_{es} \times E_{ps} \times A_{ps}$$

where: $$\varepsilon_{es} = 0.5 \times \frac{f_{co}}{E_{ci}}$$

f_{co} = the stress in the concrete adjacent to the tendon after transfer.
E_{ci} = the modulus of elasticity of the concrete at the time of transfer.

In the formula for ε_{es} given above, the factor of 0.5 takes account of the averaging effect of several tendons stressed sequentially (BS8110, Part 1, Clause 4.8.3[4]). If this is not the case, this factor may have to be modified.

Shrinkage of the Concrete

BS8110[4] covers this subject extensively in Section 4.8.4 of Part 1 and Section 7.4 of Part 2. Special care should be taken in thin members (e.g. slabs) subjected to low humidity (such as in some heated buildings) when shrinkages of more than 400×10^{-6} can occur.

The force loss is given by:

$$\delta P_{sh} = \varepsilon_{sh} \times E_{ps} \times A_{ps}$$

where: ε_{sh} = shrinkage strain of concrete

Creep of Concrete

Creep loss is based on the stress in the concrete at the level of the tendons. These losses are extensively covered by BS8110, Part 1, Section 4.8.5 and Part 2, Section 7.3[4]. They can have a very large effect in highly stressed thinner members.

The force loss is given by:

$$\delta P_{cr} = \varepsilon_{cc} \times E_{ps} \times A_{ps}$$

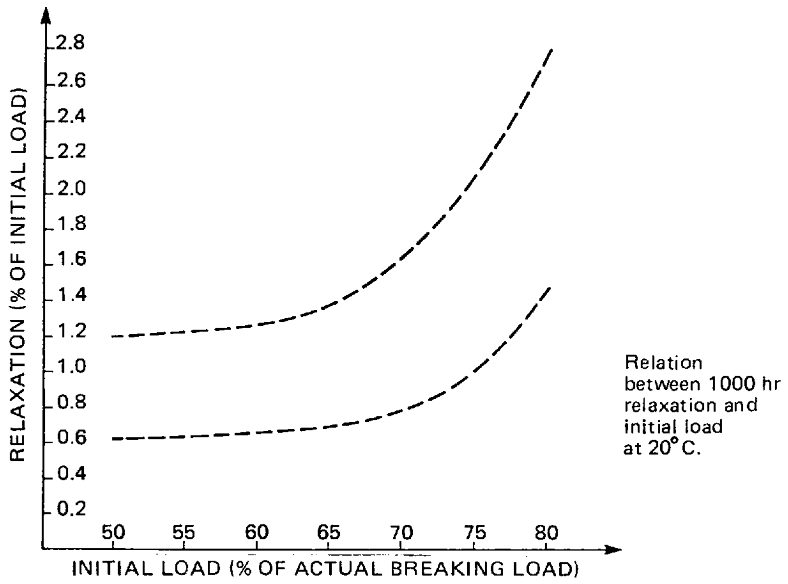
where: $$\varepsilon_{cc} = \frac{f_{co}}{E_{ci}} \times \phi$$

ϕ = the creep coefficient (BS8110, Part 2, Figure 7.1[4]).

For ribbed structures, an effective thickness should be obtained from the ratio of volume to surface area.

Relaxation of the tendons

The stress in the tendons reduces with time because of the relaxation of the steel. The amount of relaxation depends on the type of strand and the initial stress. Figure B3 illustrates typical relaxation curves for various types of strand and load levels.



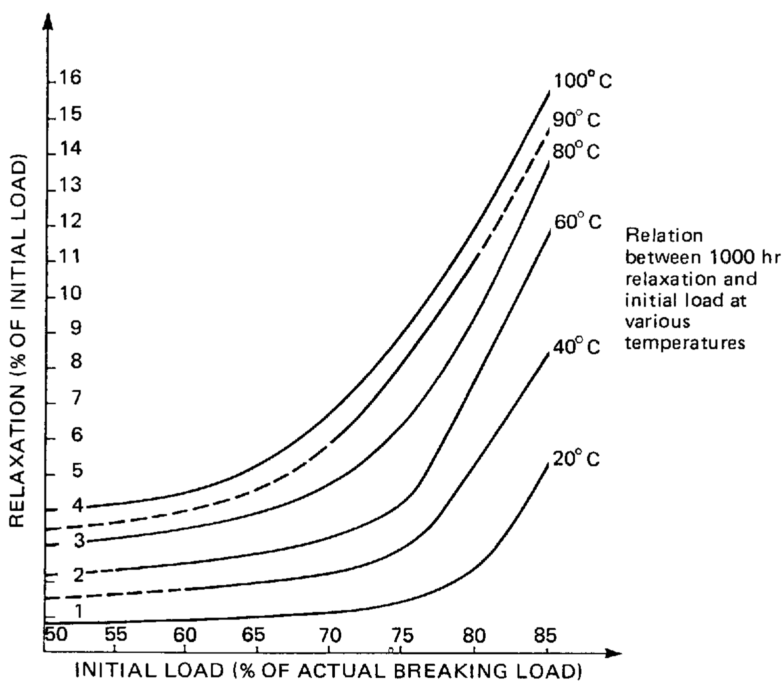


Figure B3: Relaxation curves for types of strand at various load levels

The force loss is given by:

$$\delta P_r = \text{1000-hour relaxation value} \times \text{relaxation factor} \times \text{the prestress force at transfer}$$

The 1000-hour relaxation value is given in BS5896[10] and the relaxation factor in BS8110, Part 1, Clause 4.8.2[4]. Class 2, low relaxation steel, is normally used in buildings. Data for the relaxation of this type of steel are given in Table B2.

Force at transfer as a % of characteristic strength of tendon	1000-hour relaxation	Relaxation factor	Force loss as a % of force at transfer
80%	4.5%	1.5	6.75%
70%	2.5%	1.5	3.75%
60%	1.0%	1.5	1.50%

Table B2: Relaxation for Class 2 low-relaxation steel

Note:
1. Characteristic strength of tendon = $f_{pu} \times A_{ps}$
2. The 1000-hour relaxation values from BS 5896[10] given in Table B2 above can be replaced with the manufacturer's values if available.

APPENDIX C: Calculation of Tendon Geometry

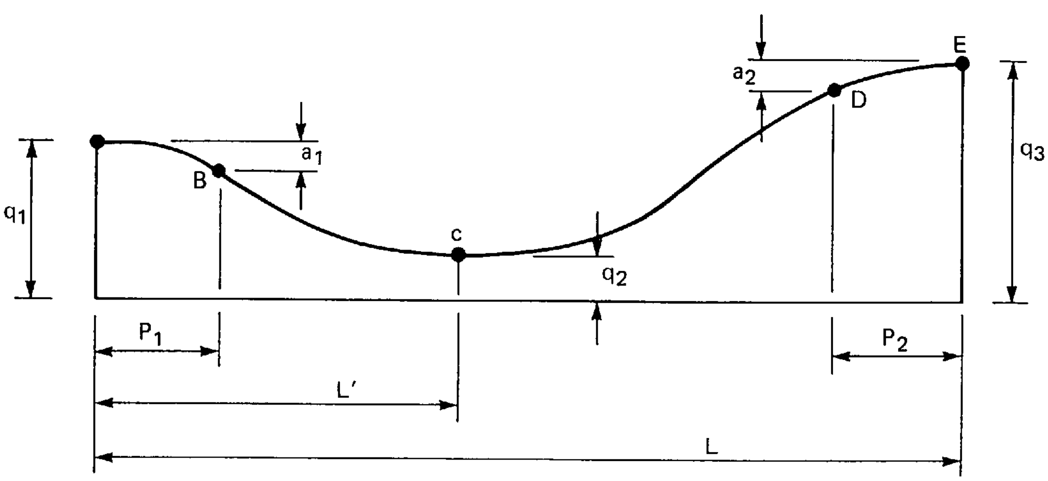


Figure C1: Tendon geometry

Consider the three parabolas AB, BCD and DE.

Parabola	AB	y	=	k_1x^2
	BCD	y	=	kx^2
	DE	y	=	k_2x^2

For parabola AB,

$$-a_1 = k_1p_1^{\,2}$$

Similarly for DE,

$$-a_2 = k_2(-p_2)^2$$

Let $Q_1 = q_1 - q_2$ and $Q_2 = q_3 - q_2$

Then for parabola BCD,

$$(Q_1 - a_1) = k(L' - p_1)^2$$

and

$$(Q_2 - a_2) = k(L - L' - p_2)^2$$

The slope of the parabolas at any point is dy/dx, and the parabolas are tangential at B and D.

For parabola AB,

$$\frac{dy}{dx} = \phi_1 = 2k_1p_1$$

similarly for DE,

$$\frac{dy}{dx} = \phi_2 = -2k_2p_2$$

As the parabolas are tangential at B and D, the slopes of the two parabolas which meet at each of these points will be equal.

For parabola BCD,

$$\frac{dy}{dx} = \phi_1 = -2k(L'-p_1)$$

and

$$\frac{dy}{dx} = \phi_2 = 2k(L-L'-p_2)$$

Using these equations it is possible to obtain expressions for k_1 and k_2 in terms of k.

$$2k_1p_1 = -2k(L' - p_1)$$

$$k_1 = \frac{-k(L' - p_1)}{p_1}$$

and

$$-2k_2p_2 = 2k(L - L' - p_2)$$

$$k_2 = \frac{-k(L - L' - p_2)}{p_2}$$

Substitute the values of k_1 and k_2 into the original equations for parabolas AB and DE.

Therefore:

$$a_1 = kp_1(L' - p_1)$$

and

$$a_2 = kp_2(L - L' - p_2)$$

Substitute the values of c_1 and c_2 into the original equations for parabola BCD.

Therefore:

$$Q_1 - kp_1(L' - p_1) = k(L' - p_1)^2$$

and

$$Q_2 - kp_2(L - L' - p_2) = k(L - L' - p_2)^2$$

Solving for k in each case

$$\frac{1}{k} = \frac{(L' - p_1)^2 + p_1(L' - p_1)}{Q_1}$$

and

$$\frac{1}{k} = \frac{(L - L' - p_2)^2 + p_2(L - L' - p_2)}{Q_2}$$

These equivalents rationalise to give the quadratic:

$$lx^2 + mx + n = 0$$

where:

$$l = (q_1 - q_3)$$
$$m = (p_2 - 2L)(q_1 - q_2) + p_1(q_3 - q_2)$$
$$n = (q_1 - q_2)(L - p_2)L$$

with the solution

$$L' = \frac{-m \pm \sqrt{(m^2 - 4ln)}}{2l}$$

Once L' has been calculated, a_1 and a_2 can be found using:

$$a_1 = \frac{(q_1 - q_2)p_1}{L'}$$

and

$$a_2 = \frac{(q_3 - q_2)p_2}{(L - L')}$$

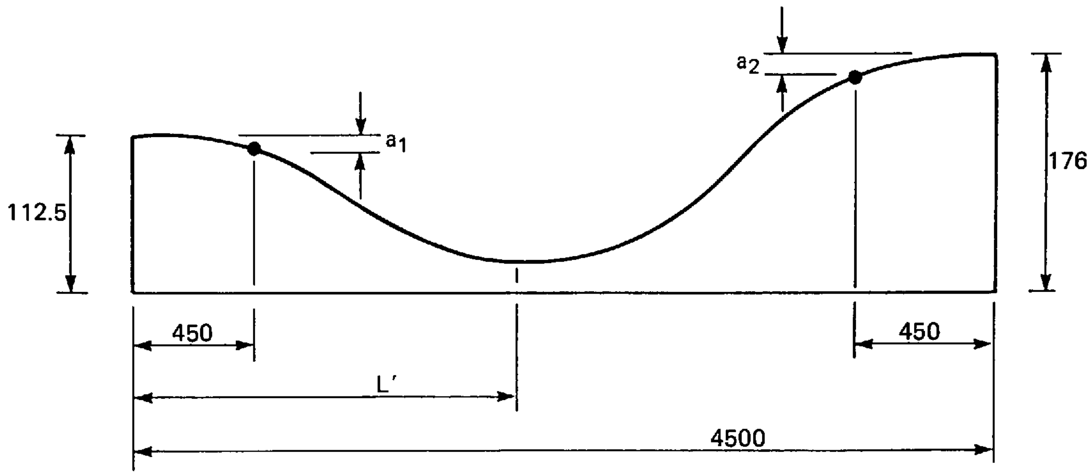


Figure C2: Solution for the transverse direction of Example A1.

For the case shown:

$l = 112.5 - 176 \quad = \quad -63.5\text{mm}$

$m = (450 - 2 \times 4500) \times (112.5 - 33) + 450 \times (176 - 33) \quad = \quad -615375$

$n = (112.5 - 33) \times (4500 - 450) \times 4500 \quad = \quad 1.449 \times 10^9$

$L' = \dfrac{615375 - \sqrt{(615375^2 + 4 \times 63.5 \times 1.449 \times 10^9)}}{2 \times -63.5} \quad = \quad 1958.75$

$a_1 = \dfrac{(112.5 - 33)}{1958.75} \times 450 \quad = \quad 18.27\text{mm}$

$a_2 = \dfrac{(176 - 33)}{(4500 - 1958.75)} \times 450 \quad = \quad 25.32\text{mm}$

$k = \dfrac{(112.5 - 33)}{(1958.62 - 450)^2 + 450 \times (1958.75 - 450)} \quad = \quad 2.69 \times 10^{-5}$

$k_1 = -2.69 \times 10^{-5} \times (1958.75 - 450)/450 \quad = \quad -9.02 \times 10^{-5}$

$k_2 = -2.69 \times 10^{-5} \times (4500 - 1958.75 - 450)/450 \quad = \quad -12.50 \times 10^{-5}$

APPENDIX D: Calculation of Secondary Effects Using Equivalent Loads

Equivalent loads can be used to represent the forces from prestress. These will automatically generate the combined primary and secondary effects when applied to the structure. Figure D1 shows the commonly occurring equivalent loads for typical prestress situations.

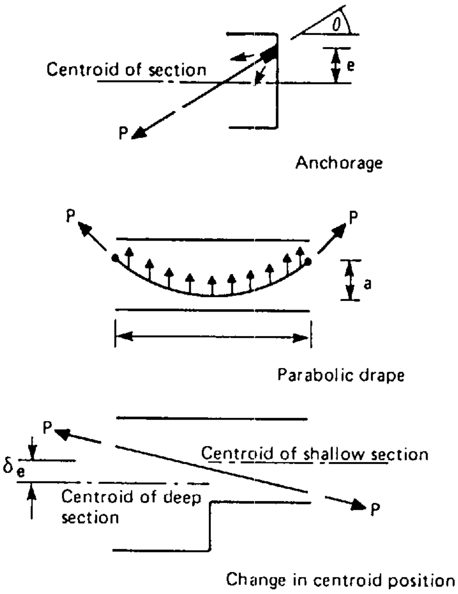


Figure D1: Commonly occurring equivalent loads

One method of separating the secondary from the primary effects is to use a frame analysis with the equivalent prestress load acting alone. The resultant moment and shear diagrams include both the primary and secondary effects. In order to obtain the secondary effects, it is only necessary to consider the moments and forces at the supports and subtract the primary effects from them. The secondary moments along each span vary linearly from end to end. This method will be known as method A.

To illustrate method A, the Ultimate Limit State for the transverse direction in Example A1 of Appendix A is used and the secondary effects obtained as follows:

1. Calculate the equivalent prestress loads in the spans using a load factor of 1.0.

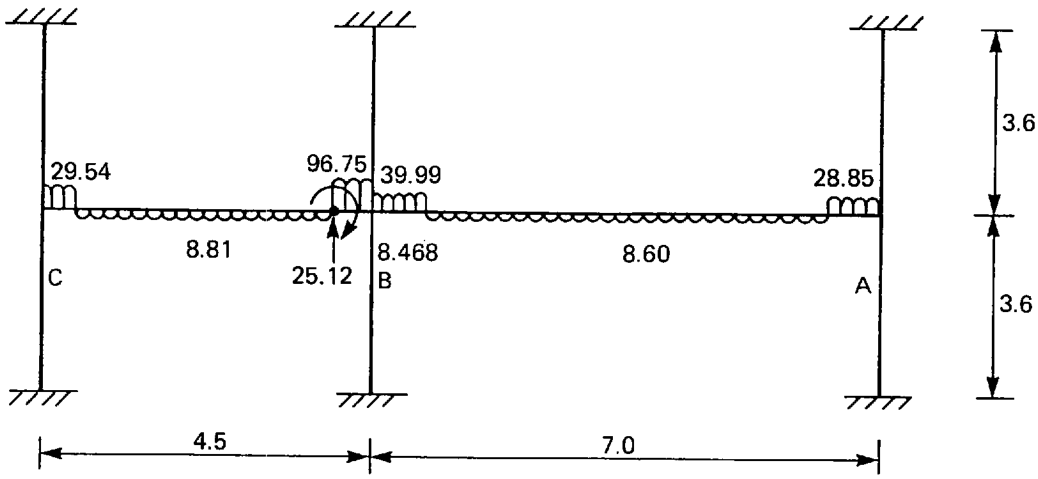


Figure D2: Equivalent balanced loads

2. Analyse the structure and obtain the bending moment diagram.

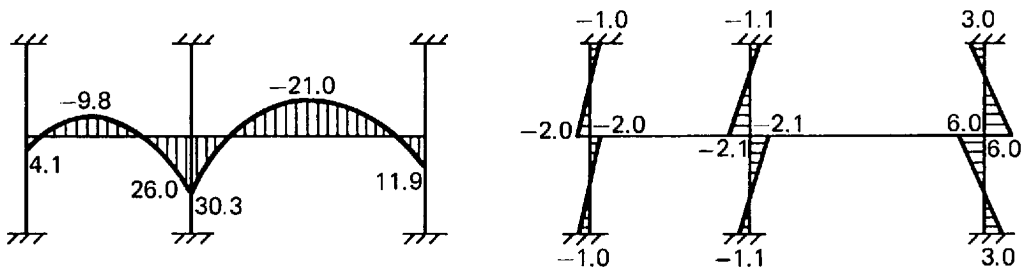


Figure D3: Moments due to primary and secondary effects

3. Calculate the primary moments due to prestress (Pe) in the slab at each support. There are no primary moments in the columns.

At support C,	Pe = 0
At support B(C),	Pe = -172 kNm
At support B(A),	Pe = -172 kNm
At support A,	Pe = 0

4. Subtract the primary moments from step (2). At this stage it should be noted that the moments and reactions in the columns from the frame analysis are due entirely to secondary effects.

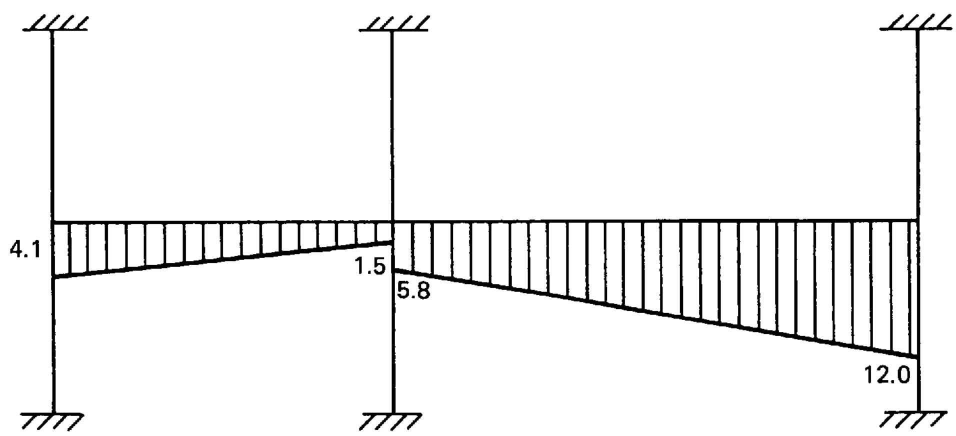


Figure D4: Bending moment diagram due to secondary effects

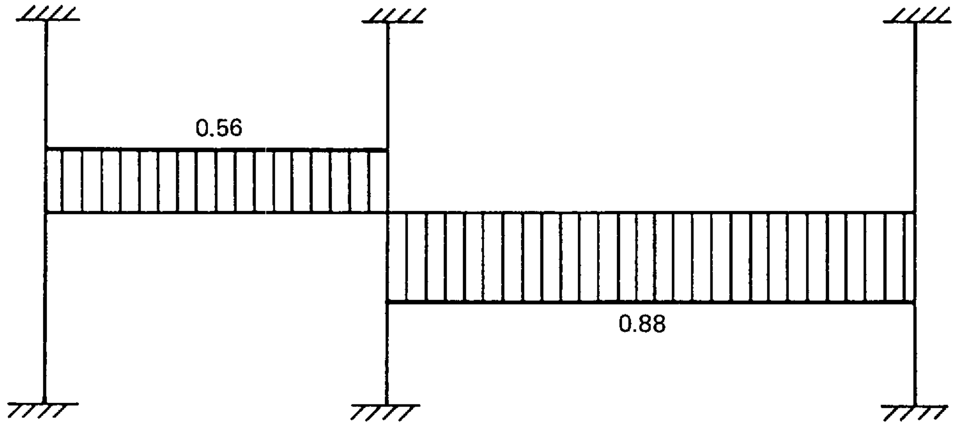


Figure D5: Shear force diagram due to secondary effects

An alternative method of calculating secondary effects is detailed below. This will be known as method B.

As there are no primary prestress forces in the columns, the column moments and reactions are entirely due to secondary effects. So the secondary effects in the slab can be easily obtained by applying these column reactions and moments to the slab as shown in Figure D6.

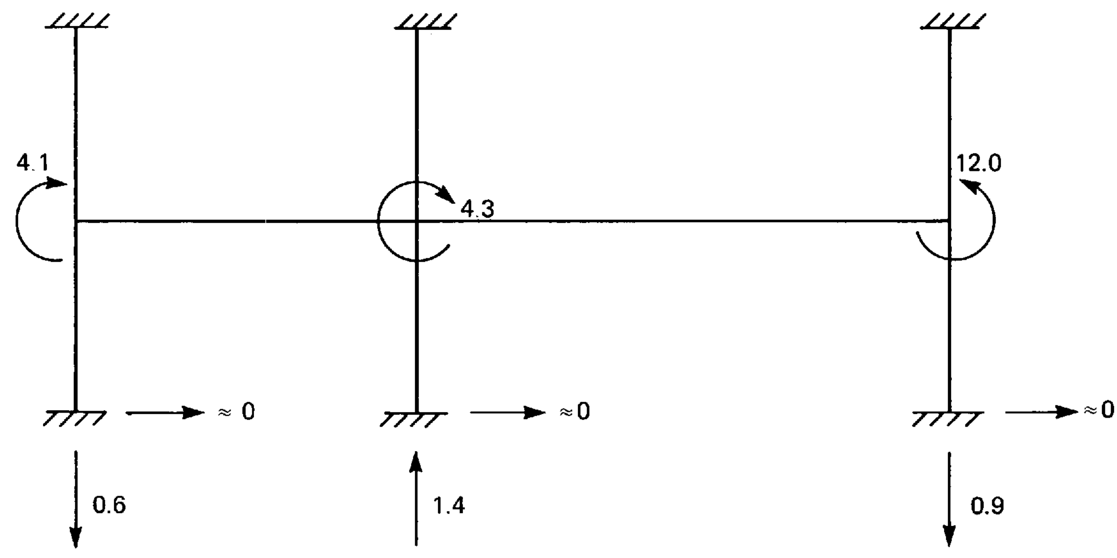


Figure D6: Column reactions and moments due to secondary forces

This results in the secondary moments and shears in the slab as shown in Figures D4 and D5.

APPENDIX E: Calculation and Detailing of Anchorage Bursting Reinforcement

In this Appendix two examples requiring bursting reinforcement are considered. For each the un-tensioned reinforcement requirement is calculated and its position in the slab detailed.

Note: The first example refers to tendons in Design Example 1 of Appendix A and the second example refers to the beam tendons in Design Example 2 of that appendix.

Example 1

Depending on the tendon layout chosen from the calculations of Design Example 1 in Appendix A, anchorages will be in groups of 1, 2, 3 or 4. The following example is for a group of 4 tendons of 12.9mm strands (unbonded) in a 225mm thick slab, as shown in Figure E1.

A group of unbonded anchorages for four 12.9mm strands in a 225mm deep slab, as shown in Figure E1.

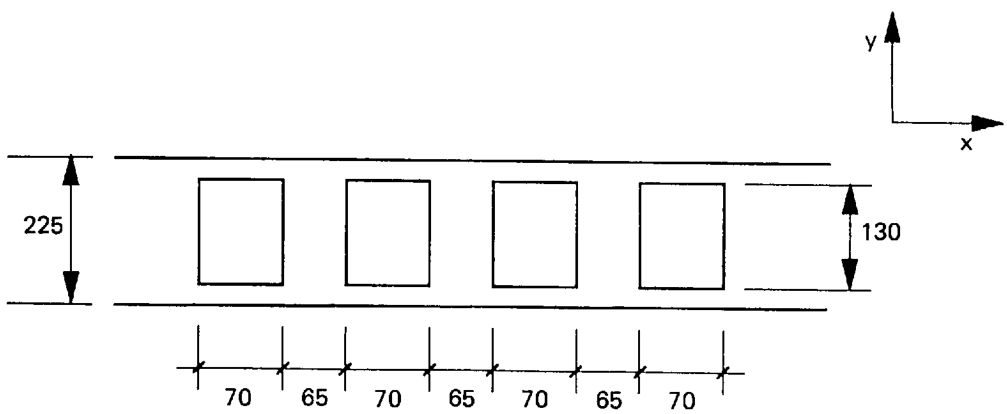


Figure E1: Anchorage layout for Example 1.

Characteristic strength of the tendon = 186 kN

$$P_o = 0.70 \times 186 \text{ kN} = 130.2 \text{ kN}$$

In the y-y direction,

$$y_{po} = 130/2 = 65 \text{ mm}$$

$$y_o = 225/2 = 112.5 \text{ mm}$$

therefore, $y_{po}/y_o = 0.58$

From Table 4.7[4]

$$F_{bst}/P_o = 0.146$$

$$F_{bst} = 0.146 \times 130.2 = 19\text{kN}$$

Therefore, at Serviceability Limit State with reinforcement acting at a stress of 200 N/mm^2,

$$A_s = \frac{19 \times 10^3}{200} = 95\ mm^2/anchorage$$

At Ultimate Limit State, $A_s = \dfrac{186 \times 0.146 \times 10^3}{0.87 \times 460} = 68\ mm^2/anchorage$ (use 2T10)

This un-tensioned reinforcement should be between 22.5 and 225 mm from the anchor-bearing face.

In the x-x direction,

$$x_{po} = (4 \times 135)/2 = 270\ mm$$

$$x_o = \text{unlimited (say 1500 mm)}$$

Therefore, $x_{po}/x_o = 0.18$

From Table 4.7[4]

$$F_{bst}/P_o = 0.23$$

$$F_{bst} = 4 \times 130.2 \times 0.23$$

$$= 120\ kN$$

Therefore,

$$A_s = \frac{120 \times 10^3}{200} = 600\ mm^2\ \text{(use } 2 \times 4T12)$$

This un-tensioned reinforcement should be between 150 and 1500 mm from the anchorage bearing face.

Figure E2 shows the practical detailing of these requirements.

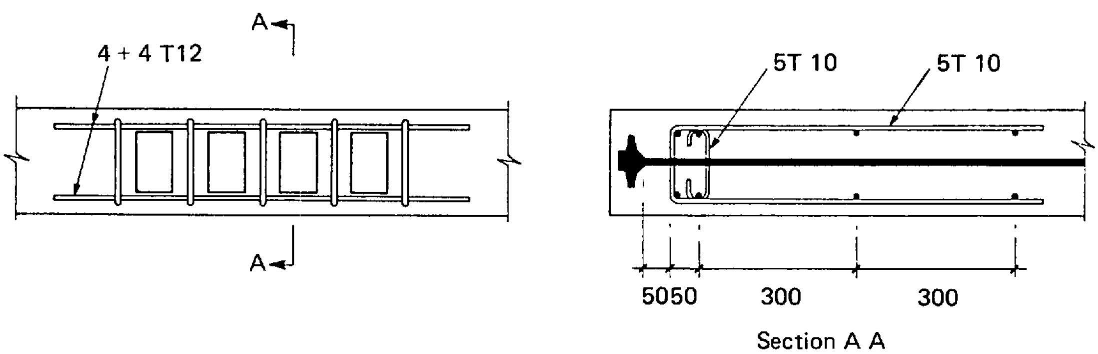


Figure E2: Bursting reinforcement distribution for Example 1.

Comment: It is not usually required to do an equilibrium study for flat plates with regularly spaced tendons, provided they are stressed in such a sequence as to avoid problems at corners.

Example 2

The ribs in Example 2 of Appendix A have unbonded anchorages and their bursting design will be similar to Example 1 above. The design of the anchorage bursting reinforcement for the bonded tendons in the beams is outlined below.

The design requires eleven 15.7 mm bonded strands in the beam. It is decided to use two tendons of four strands each and one of three strands, with the anchorages arranged as in Figure E3.

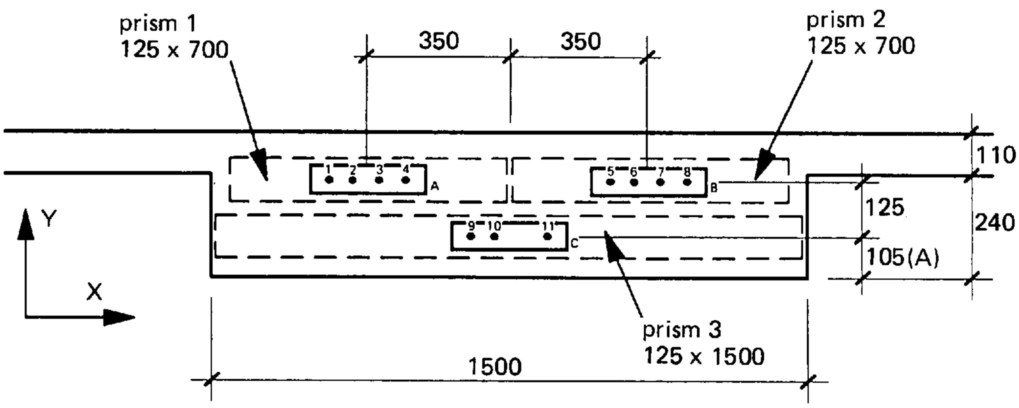


Figure E3: Anchorage layout for Example 2.

The anchorages are positioned so that the c.g.s of the tendons corresponds to the centre of gravity of the concrete (c.g.c.); from the calculations in Example 2, this is 196mm above the beam soffit.

Hence, assuming the arrangement in Figure 3,

8(strands) $\times$ (125 + A) + 3(strands) $\times$ A = 11 $\times$ 196
i.e. 1000 + 8A + 3A = 2156

hence A = 105 mm.

For the stressing sequence, it is assumed that one strand in each tendon is stressed until all strands in all three tendons have been stressed (i.e. if the strands are numbered 1-11 as shown in Figure 3 the stressing sequence would be 1, 5, 9, 4, 8, 11, 2, 6, 10, 3 and 7). In this way, there is no need to consider intermediate stages and it is likely to give the least amount of bursting reinforcement.

In order to check the end block fully, two individual checks are required, namely:

a) Single anchorage bursting
b) End block stability

a) Figure E4 shows how the end block can be divided into individual end blocks or prisms for each anchorage. These must be rectangular and symmetrical.

Prisms for anchorages A and B are 125 deep × 700 wide. The prism for anchorage C is 125 deep × 1500 wide.

Anchorages A and B
x-x direction

$x_{po}/x_o \quad = \quad 137.5/350 = 0.39$

The jacking force per strand is 185.85 kN
Hence $P_o \quad = \quad 4 \times 185.85 = 743.4$ kN

From BS 8110[4], Table 4.7

$F_{bst}/P_o \quad = \quad 0.203$

Hence $F_{bst} \quad = \quad 0.203 \times 743.4 = 150.9$ kN
and the reinforcement required at the allowable stress of 200 N/mm² is

$A_s \quad = \quad 150.9 \times 10^3/200 = 754.5 \text{ mm}^2$
positioned between $0.2x_o$ to $2.0x_o$
i.e. 70 to 700mm from anchor.

Similar calculations for the y-y direction of anchorages A and B and both directions of anchorages C (three strands only) yield the following reinforcements:

Anchorage A and B
y-y direction

$A_s \quad = \quad 409 \text{ mm}^2$ at 12.5 to 125 mm

Anchorage C
x-x direction

$A_s \quad = \quad 641 \text{ mm}^2$ at 150 to 1500 mm

y-y direction

$A_s \quad = \quad 307 \text{ mm}^2$ at 12.5 to 125mm.

b) Overall stability in y-y direction [21]

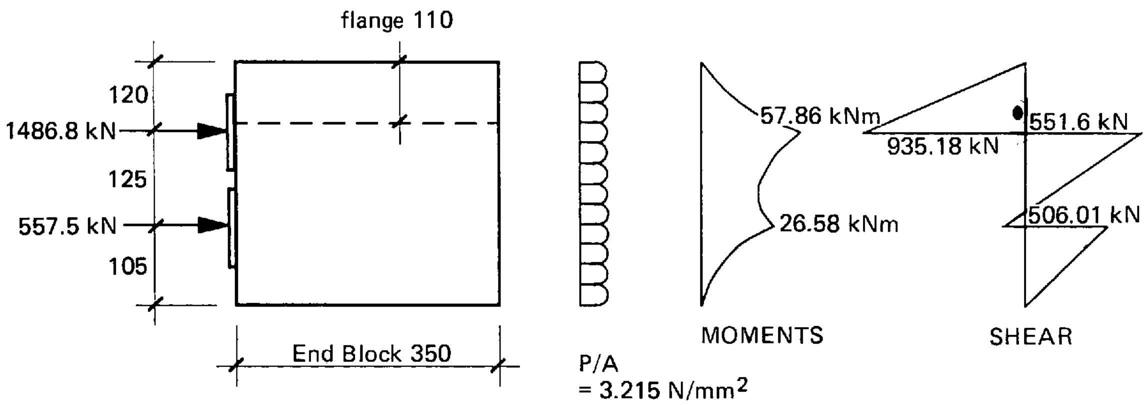


Figure E4: End block moments and forces: y-y direction.

Because anchorages forces are increased evenly as explained above, and the anchorages are located on the c.g.c., the stress block behind the anchorages is uniform and equal to

$11 \times 185.85 \times 10^3/0.6359 \times 10^6 \quad = \quad 3.215\ \text{N/mm}^2$

(0.6359×10^6 is the area of the section from Example A2)

The moments can be calculated thus:

$M_A = 3.215 \times 1500 \times 230^2 \times ½ \times 10^{-6} - 557.55 \times 125 \times 10^{-3} \quad = \quad 57.86\ \text{kNm}$

$M_B = 3.215 \times 1500 \times 105^2 \times ½ \times 10^{-6} \quad = \quad 26.58\ \text{kNm}$

Hence, for the maximum moment of 57.68 kNm

and a lever arm of ½ block length (= 175 mm),

steel required $A_s = \dfrac{57.86}{0.175 \times 0.200} \quad = \quad 1653\ \text{mm}^2$

distributed over distance of 175mm to 350mm from the anchorage faces.

From reference 21, minimum steel

$= 0.3\% \times 1500 \times 350 \quad = \quad 1575\ \text{mm}^2$, which is satisfactory.

Similarly in the x-x direction

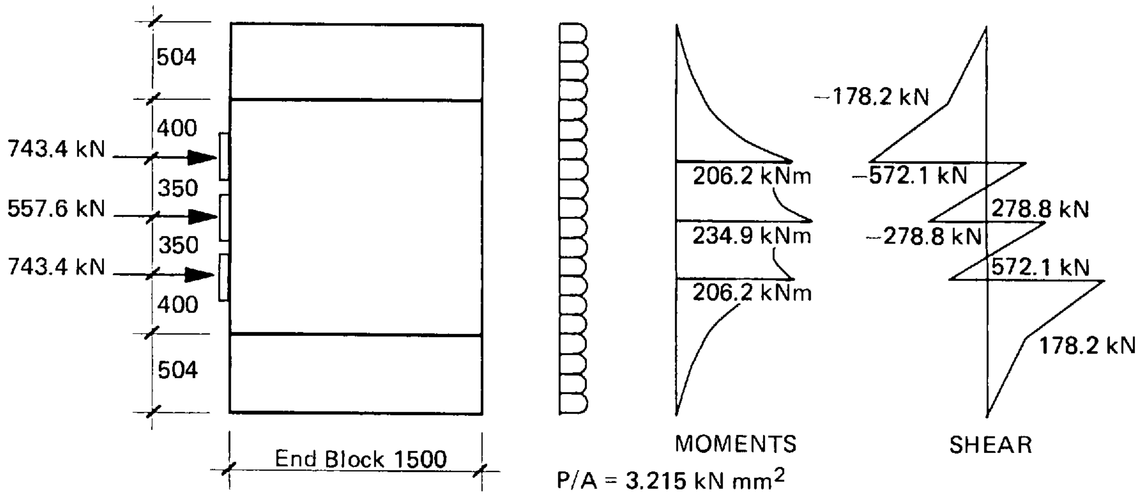


Figure E5: End block moments and forces: x-x direction.

$M_A = M_C \quad = \quad 3.215 \times 110 \times 504 \times (400 + 504/2) \times 10^{-6}$

$\qquad + 3.215 \times 350 \times 400^2 \times ½ \times 10^{-6} \quad = \quad 206.2$ kNm

$M_B \quad = \quad 3.215 \times 110 \times 504 \times (750 + 504/2) \times 10^{-6}$
$\qquad + 3.215 \times 350 \times 750^2 \times ½ \times 10^{-6}$
$\qquad - 743.4 \times 350 \times 10^{-3} \quad = \quad 234.9$ kNm

Hence,

$A_s \quad = \quad 234.9/(0.75 \times 0.200) \quad = \quad 1566\ mm^2$

Minimum steel
$= 0.3\% \times 350 \times 1500 \quad = \quad 1575\ mm^2$
distributed over distance of 750 to 1500mm from the anchorage faces

Note: The above moments are slightly overstated since the anchorage force has been assumed (conservatively) to be a point load.

Flow of stress into flange[21]
Load in flange $= 3.215 \times 2508 \times 110 \times 10^{-3} = 887$ kN

Width of web = 1500 mm

$y_{po}/y_o = 1500/2508 = 0.60$

From BS 8110[4]
$F_{bst}/P_o = 0.14$

therefore F_{bst} = 124 kN

Required area of steel = 620 mm² distributed over distance 250 to 2500 mm from the anchorage faces.

Check on horizontal shear capacity.

From Figure E4 , maximum shear force = 935.2 kN

giving a shear stress of $\dfrac{935.2 \times 10^3}{1500 \times 350}$ = 1.78 N/mm²

$\ngtr$ 2.25 N/mm² therefore fine.

Check on vertical shear capacity.

From Figure E5, maximum shear force = 572.1kN

giving a shear stress of $\dfrac{572.1 \times 10^3}{1500 \times 350}$ = 1.09 N/mm²

$\ngtr$ 2.25 N/mm² therefore fine.

In the flange area, maximum shear force = 178.2 kN

giving a shear stress of $\dfrac{178.2 \times 10^3}{1500 \times 110}$ = 1.08 N/mm² therefore fine.

The reinforcement layout given in Figure E6 satisfies all the preceding bursting and end-block stability requirements.

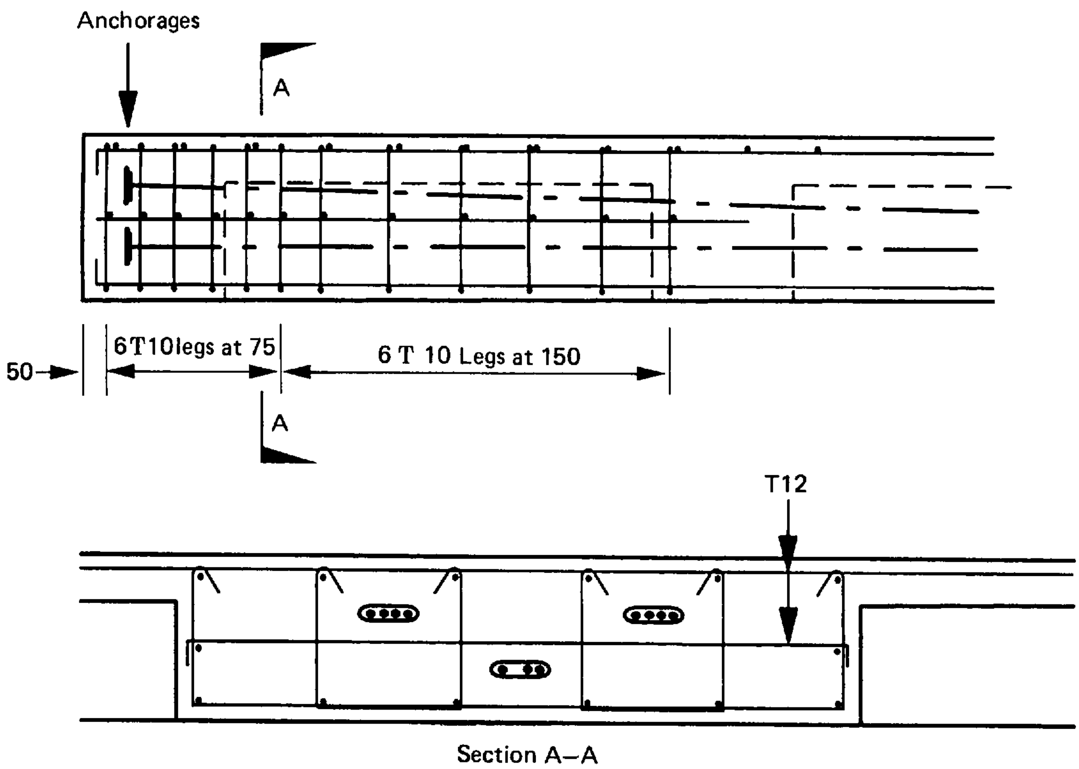


Figure E6: Layout of end block reinforcement.

APPENDIX F: Simplified Shear Check: Derivation of Figures 17 and 18

See BS 8110 Part 1[4].

Assumptions:

1. Charts are drawn for internal columns

2. $V_c = v_c \times u \times d/1000$ in kN
   where v_c is shear resistance of the concrete (N/mm^2)
   u is the length of the critical perimeter (mm)
   d is the effective depth (mm)

3. $d \approx 0.9h$
   where h is the depth of slab

4. Columns are square of dimension c

5. $u = 4(c + 3d) = 4(c + 2.7h)$

6. Loading is uniformly distributed.
   Ult. Load = 1.5 × (Char. Dead Load+Char. Total Imp. Load)
   where Char. Total Imp. Load, Q_T = Live Load+Finishes

7. Concrete density = 24 kN/m^3

8. Applied shear force $V \approx 1.5\ A\ (24h/1000 + Q_T)$ in kN
   where A is the appropriate area of floor in m^2

A *Check at first critical perimeter (Figure 17)*

$$V_c = v_c \times u \times d/1000$$
$$= 3.6\ hv_c(c + 2.7h)/1000\ kN$$

$$V \leq V_c$$

Therefore $1.5(24h/1000 + Q_T) \times A \leq 3.6hv_c(c + 2.7h)/1000$

$$Q_T \leq 12v_ch(c + 2.7h)/5000A - 24h/1000\ kN/m^2$$

B *Check at face of column (Figure 18)*

Assume $f_{cu} = 40\ N/mm^2$

Maximum design shear strength $= 5\ N/mm^2$ or $0.8\ \sqrt{f_{cu}}\ N/mm^2$
$= 5\ N/mm^2$

$$V_{cmax} = 4c \times 0.9h \times 5/1000\ kN$$
$$= 18ch/1000\ kN$$

$$V \leq V_{cmax}$$

$$1.5A\ (24h/1000 + Q_T) \leq 18ch/1000$$

$$Q_T \leq 12ch/1000A - 24h/1000\ kN/m^2$$

APPENDIX G: Vibration of Post-tensioned Concrete Floors

The SCI publication 'Design Guide on the Vibration of Floors'[22] was specifically written for composite steel/concrete floors. Post-tensioned flat slabs usually have greater mass and a lower fundamental natural frequency than slabs on profiled metal decking. Although increasing the mass of the floor reduces the dynamic response, floors of lower natural frequency are excited by much larger components of the walking force. It is therefore likely that dynamic response will control the thickness of some flat slabs.

Most of the SCI guide is equally relevant to other forms of floor construction. The main difficulty in using the guide for flat slabs is estimating their effective widths and spans. The values for these dimensions are given in terms of beam and slab stiffnesses. The values cannot easily be modified for the various configurations of flat slabs.

The design procedure given in the SCI guide only considers the fundamental frequency of the floor. However only part of the mass of the floor is considered as taking part in the response. This area is defined by the effective width and span. An alternative approach is to consider all the natural frequencies and the total mass of the floor. In practice, only a limited number of natural frequencies need to be considered because the contribution to the total response from the higher frequencies is minimal.

The procedure given below uses this second approach. The dynamic response of the fundamental frequency is calculated. This is then multiplied by a factor to give the total dynamic response.

There are problems with the SCI guide in the transition between high- and low-frequency floors. There can sometimes be a jump in the calculated response of a factor of ten. The procedure below gives functions which lead to a stepless transition. It also gives expressions for estimating the fundamental natural frequency of flat slabs, including the cases where there are no perimeter beams.

The dynamic responses of floors have been checked to the deflection span/depth ratios determined by deflection considerations. For normal office space, the responses fall within the acceptance criteria, provided additional requirements on the minimum number of bays and slab thicknesses are met. Table 1 includes these additional requirements.

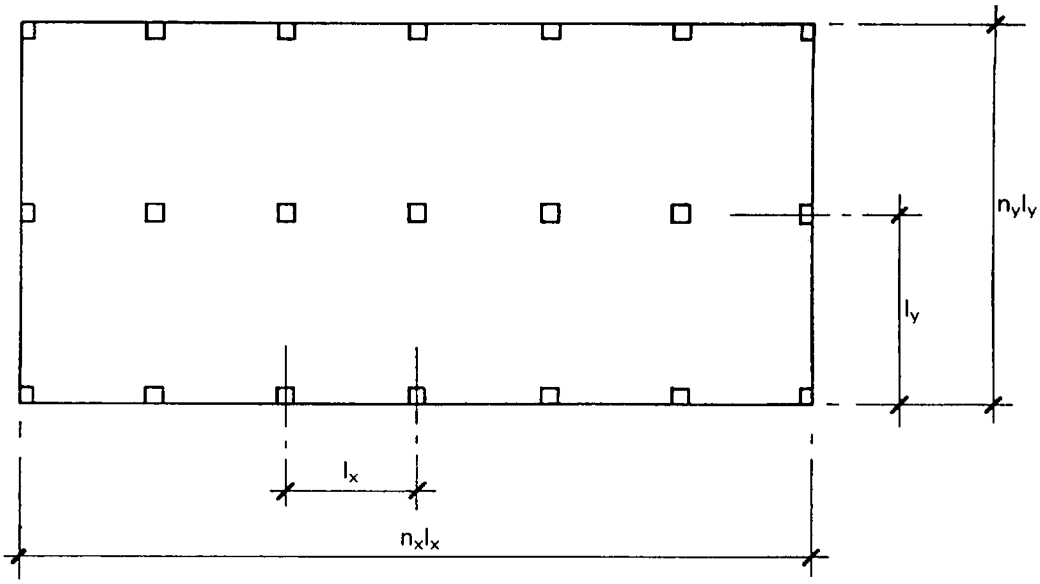
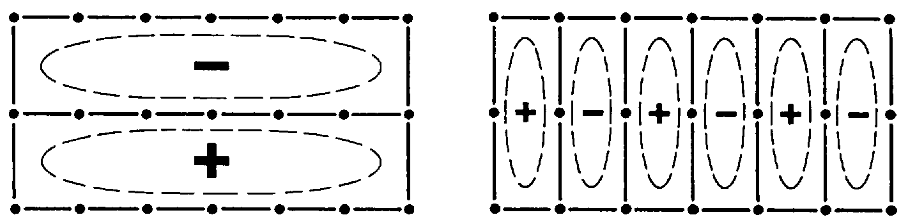


Figure G1:

Two families of vibration modes are considered. This is conservative because some of the higher harmonics of the two families are in common.



(a) x family (b) y family

Figure G2: Families of vibration modes

x family	y family
Effective aspect ratio = λ_x	Effective aspect ratio = λ_y
(1) $\lambda_x = \frac{n_x l_x}{l_y}\sqrt[4]{\frac{EI_y}{EI_x}}$	$\lambda_y = \frac{n_y l_y}{l_x}\sqrt[4]{\frac{EI_x}{EI_y}}$

EI_x is flexural stiffness of slab spanning in the x direction per unit width (Nm^2/m)

EI_y is flexural stiffness of slab spanning in the y direction per unit width (Nm^2/m)

(2)

$$f_x' = k_x \frac{\pi}{2} \sqrt{\frac{EI_y}{ml_y^4}} \qquad f_y' = k_y \frac{\pi}{2} \sqrt{\frac{EI_x}{ml_x^4}}$$

k_x and k_y depend on the effective aspect ratio.
m is mass per unit area (t/m^2) (dead load + 10% live load)

Let "r" represent "x" or "y" as appropriate

$$k_r = 1 + \frac{1}{\lambda_r^2}$$ for solid or coffered slabs

$$k_r = \sqrt{1 + \frac{1}{\lambda_r^4}}$$ for ribbed slabs

For slabs with perimeter beams:

$$f_r = f_r'$$

For slabs without perimeter beams:

$$f_b = \frac{\frac{\pi}{2}\sqrt{\frac{EI_x}{ml_x^4}}}{\sqrt{1 + \frac{EI_x l_y^4}{EI_y l_x^4}}} = \frac{\frac{\pi}{2}\sqrt{\frac{EI_y}{ml_y^4}}}{\sqrt{1 + \frac{EI_y l_x^4}{EI_x l_y^4}}}$$ for solid or coffered slabs

Note: f_b is natural frequency of a single panel supported at its four corners

$$f_b = \frac{\frac{\pi}{2}\sqrt{\frac{EI_x}{ml_x^4}}}{\left(1 + \left(\frac{EI_x l_y^4}{EI_y l_x^4}\right)^{\frac{2}{3}}\right)^{\frac{3}{4}}} = \frac{\frac{\pi}{2}\sqrt{\frac{EI_y}{ml_y^4}}}{\left(1 + \left(\frac{EI_y l_x^4}{EI_x l_y^4}\right)^{\frac{2}{3}}\right)^{\frac{3}{4}}}$$ for ribbed slabs

$$f_r = f_r' - (f_r' - f_b)\left(\frac{1/n_x + 1/n_y}{2}\right)$$

(3) $$N_r = 1 + (0.5 + 0.1 \ln \zeta)\lambda_r$$ for solid or coffered slabs

$$N_r = 1 + (0.65 + 0.1 \ln \zeta)\lambda_r$$ for ribbed slabs

ζ is critical damping ratio $\approx$ 0.02 for open plan offices

(4) $$C_r = \frac{224.8}{f_r^2 \zeta} \qquad f_r \le 3\text{Hz}$$

$$C_r = \frac{27.2}{\zeta} \qquad 3\text{Hz} \le f_r \le 4\text{Hz}$$

$$C_r = \frac{83.2 - 14 f_r}{\zeta} \qquad 4\text{Hz} \le f_r \le 5\text{Hz}$$

$$C_r = \frac{0.88\,(20 - f_r)}{\zeta} + 2(f_r - 5) \qquad 5\text{Hz} \le f_r \le 20\text{Hz}$$

$$C_r = 30 \qquad 20\text{Hz} \le f_r$$

(5) $$R_r = \frac{C_r N_r}{m n_x n_y l_x l_y}$$

$$R = R_x + R_y$$

Where $R \le 4$ Special office
$R \le 8$ General office
$R \le 12$ Busy office

APPENDIX H: Advertisements

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Oasys **ADPRSL**

ADPRSL is part of the Oasys ADC suite of computer programs for the analysis and design of reinforced and prestressed concrete elements ADPRSL addresses post-tensioned flat slabs with bonded or unbonded tendons. The program was written to follow the requirements of BS 8110 Part 1 and Part 2, Concrete Society Technical Reports No 17 and 25, and the draft version of a new Concrete Society Design Manual on Post-tensioned Concrete Floor Design.

- *The manual design of continuous slabs in post-tensioned concrete can involve hours of arduous effort to determine an efficient tendon profile and prestressing force. ADPRSL automates this process to produce designs that are economic and efficient.*
- *Numerous options have been incorporated into the program to meet the needs of a wide variety of design problems.*
- *Following analysis the user may explore other prestress and balanced load combinations quickly.*
- *ADPRSL will also provide information on quantities of materials used so allowing comparisions with other forms of construction.*
- *The program is currently for single-way analysis and design, and the user must consider both frame directions of the structure separately. ADPRSL uses the 'Load Balancing Method' to carry out the design of rectangular slabs and Tee-sections, for one-way and two-way spanning floors.*
- *Up to 10 spans, including end cantilevers may be considered, and rectangular drops at columns can be simulated.*
- *Columns are considered as fixed in position and in direction at their remote ends. The structure is analysed using the Equivalent Frame Method. In this method the structure is analysed as a frame in one direction, with the slab considered to be a continuous strip of width equal to the panel width.*
- *The use of equivalent loads automatically takes into account the secondary (parasitic) effects in the structure.*

The initial input information required consists of the slab and column geometry and design loads. An ultimate analysis is then carried out without redistribution. After data checking ADPRSL requests tendon data, global loss factors, deflection limits, material properties and balanced load details and then carries out the prestressed analysis and design wherever relevant default values are provided. Serviceability and Ultimate Limit State (ULS) moments can be reduced to the column face if required, and dead and live load percentages set for the transfer condition. It is also possible to individualize prestress forces in any spans and set a minimum prestress level.

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Oasys **AP+ADC**

THE CONCRETE SOCIETY

Founded in 1966 The Concrete Society brings together all those with an interest in concrete to promote excellence in its design, construction and appearance, to encourage new ideas and innovations and to exchange knowledge and experience across all disciplines.

TECHNICAL DEVELOPMENT CENTRE

The Concrete Society is a centre of excellence for technical development of concrete, producing state-of-the-art reports, recommendations and practical guides, and initiating and undertaking R&D where appropriate.

The work of the Centre is under the direction of the Technical Development Board, which includes senior personnel from all sides of the industry. The Technical Management Committee and its supporting specialist groups and working parties carry out the work of the Centre, with the help of the Technical Manager and his staff. The Centre collaborates with other overseas Concrete Societies in mutually beneficial programmes of technical development.

CONFERENCES AND EXHIBITIONS

The Society organises national and international conferences and exhibitions including DTI-supported Joint Venture exhibitions at major international events worldwide.

The Society's annual 'Concrete Day' has become a major national event in the calendar of the construction industry.

REGIONS & CLUBS

The Society is organised into 22 regions and clubs which each arrange a comprehensive programme of technical and social events in their areas. As well as all the technical benefits, the Society provides an ideal social forum for members to make valuable business and personal contacts.

CONCRETE

Members in all categories are sent a copy of each issue of CONCRETE, the Society's journal, providing up-to-date information on all aspects of concrete including design, materials, construction techniques, quality control, equipment, maintenance and repair.

CONCRETE ADVISORY SERVICE

The Concrete Advisory Service provides prompt impartial technical advice on concrete and related matters to subscribing Members of any discipline. The regionally based advisory staff are all Chartered Engineers and have wide experience in all aspects of the use of concrete and of providing appropriate advice.

Any member of staff of a subscribing group Member can tap into this vast reservoir of information on materials, technology and practice. In addition to telephone advice, visits to Member's offices or sites to discuss technical matters or solve problems can also be arranged. Reports can be provided where applicable.

AWARDS

Awards which ensure that excellence in concrete is publicly acknowledged are made for completed structures in building, civil engineering and mature structures.

MEMBERSHIP

There are two kinds of Membership.

Group Membership for firms, partnerships, government departments, local authorities, educational establishments etc.

Personal Membership giving Concrete Society benefits to individuals.

ENTITLEMENT TO USE DESIGNATORY LETTERS

Members who have appropriate knowledge and experience may apply for the qualifying grades 'Member' (MCS) and 'Fellow' (FCS).

REGIONAL ADVISORY ENGINEERS

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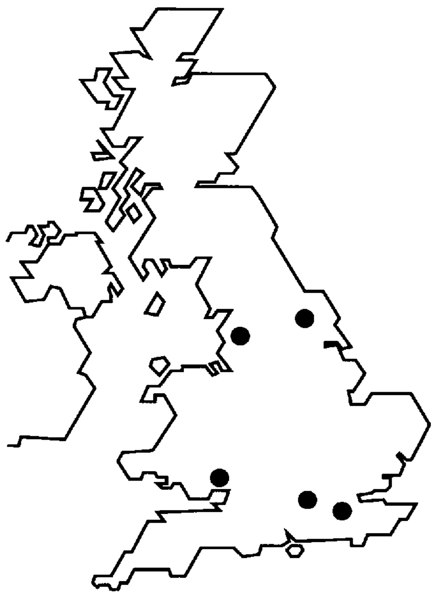

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