DEPARTMENT OF MECHANICAL ENGINEERING

ME6505 DYNAMICS OF MACHINES

UNIT NOTES

UNIT – I - Force Analysis

D'Alembert's principle states that the inertia forces and torques, and the external forces and torques acting on a body together result in statical equilibrium. In other words, the vector sum of all external forces acting upon a system of rigid bodies is zero. The vector sum of all external moments and inertia torques acting upon a system of rigid bodies is also separately zero.

The **principle of super position** states that for linear systems the individual responses to several disturbances or driving functions can be superposed on each other to obtain the total response of the system.

The **inertia force** is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude, but opposite in direction.

The **inertia torque** is an imaginary torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but opposite in direction.

1. The lengths of crank and connecting rod of a horizontal engine are 200mm and 1m respectively. The crank is rotating at 400rpm. When the crank has turned through 30^0 from the inner dead centre, the difference of pressure between cover and piston rod is $0.4N/m^2$. If the mass of the reciprocating parts is 100kg and cylinder bore is 0.4m, the calculate: (i) inertia force, (ii) force on piston, (iii)piston effort, (iv) thrust on the sides of the cylinder walls, (v) thrust in the connecting rods and (vi) crank effort.

Given: r=200mm =0.2m, l=1m, N=400rpm, $\theta=30^0$, $p1-p2=0.4N/m^2=0.4x10^6 N/m^2$,

mR=100kg, D=0.4m.

Solution: $\omega=2\pi N/60 = 2\pi \times 400/60=41.89$ rad/s

(i) Inertia force (F_I) :

$F_I= mR\ \omega\ r (\cos\theta+\cos2\theta/n) = 100 (41.89)^2 0.2 (\cos30^0+\cos2x30^0/5)$

F_I= 33.903kN ans

(ii) Net load on the piston(F_L):

$F_L= (p1-p2)A = (0.4x10^6)x\ \pi/4(0.4)^4$

F_L=50.265kN ans

(iii) Piston effort(Fp):

$Fp=F_L-F_I = 50.265\ x10^3 – 33.903x10^3$

Fp = 16.36kN ans

(iv) Thrust on the sides of the cylinder walls (F_N)

$F_N = Fp \tan\phi$

$\sin\phi = \sin\theta/n = \sin30^0/5 = 0.1$

$\phi = 5.74^0$

$F_N = Fp \tan\phi\ \ 16.36 \tan5.74$ = **1.644kN ans**

(v) Thrust in the connecting rod (F_Q)

F_Q=Fp/cos ϕ = 16.36 x10^3/cos5.74 = **16.444kN ans**

(vi) Crank effort (T):

T =F_T xr

$F_{T=}$ F_Q sin (θ+ ϕ) = 16.44 x10^3 sin (30^0+5.74^0) = 9.605kN

T =F_T x r = 9.605 x10^3 x 0.2 = **1921.13kN ans**

2. A single cylinder vertical engine has bore of 300mm and a stroke of 400mm. the connecting rod is 1000mm long. The mass of the reciprocating parts is 140kg. on the expansion stroke with the crank at 30° from the top dead centre, the gas pressure is 0.7Mpa. if the engine runs at 250 rpm determine: i) net force acting on the piston ii) resultant load on the gudgeon pin, iii) thrust on the cylinder walls, and iv) the speed above which other things remaining same, the gudgeon pin loads would be reversed in direction.

Given: D=300mm=0.3m, L=400mm=0.4m, r=04/2=0.2m, l=1000m, mR= 140kg, p=0.7x10^6 N/m^2, N=250rpm.

Solution: ω=2πN/60 = 2π x 250/60=26.18 rad/s

n/r-=1/0.2=5

(i) Net force acting on the piston (Fp):

Fp=F_L-F_I

F_L=px π/4$(D)^2$ = 0.7 x10^6 (π/4 x0.3^2) = 49480.1 N

Inertia force on the piston is given by

F_I= mR ω r (cosθ+Cos2θ/n) = 140 $(26.18)^2$0.2 (cos30^0+cos2x30^0/5) = 18538.98 N

Fp= F_L- F_I = 49480.1 – 18538.98 = **32314.52 N ans**

(ii) Resultant load on the gudgeon pin (F_Q):

F_Q= Fp/ cos ϕ

sin ϕ = sin θ/n = sin30^0/5 = 0.1

ϕ = 5.74^0

F_Q = 32314.52/cos 5.74 **= 32477.3 N ans**

(iii) Thrust on the cylinder walls (F_N):

F_N = Fp tanϕ

F_N = 32314.52 tan 5.74 = **3247N ans**

(iv) The speed above which the gudgeon pin load would be reversed in direction:

F_I $\geq$ force due to gas pressure + mR.g

mR ω_1 r (cosθ+Cos2θ/n) = 50.853.5

140 $(\omega_1)^2$0.2 (cos30^0+cos2x30^0/5) 50.853.5

$(\omega_1)^2 = 1880$

$\omega_1 = 43.36$ rad/s

$\omega_1 = 2\pi N_1/60$

$N_1 = (60 \times 43.36) / 2\pi$

$N_1 = 414$ rpm ans

3. The turning moment curve for an engine is represented by the equation, T = (20000+9500 sin2θ-5700cos2θ) N-m, where θ is the rotation of the crank. If the resisting torque is constant, find (i) Power developed, (ii)Moment of inertia of the flywheel and (iii) angular acceleration of the flywheel at 45^0 of crank rotation from IDC. The speed of engine is 180 rpm and total fluctuation of speed is 1%.

Given data:

T= (20000+9500 sin 2θ – 5700 cos2θ) N-m
N = 180rpm
ω =
18.85rad/sec
$\theta = 45^0$
C_S = 1% or 0.01

Solution :

Power developed by the engine work done per revolution = $\int_0^{2\pi} T d\theta$

$$\int_0^{2\pi} (20,000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\, d\theta$$

= 40,000 Π N-m

Tmean = workdone per revolution / 2Π = 20,0000 N-
m P = T mean× ω = 377 KW.

Moment of inertia of flywheel (I) kg-m2

T = Tmean

(20000+9500 sin 2θ – 5700 cos2θ) = 20000
9500 sin 2θ – 5700 cos2θ = 20000 – 20000
9500 sin 2θ = 5700
cos2θ sin 2θ / cos2θ =
5700 / 9500 tan 2θ = 0.6
2θ = tan-
1 0.6 2θ
= 30.96
θ = 30.96 / 2
$\theta_B = 15.48^0$. ($\theta_B = 15.5$, $\theta_C = 90 + 15.5 = 105.5^0$)

$\theta_B = 15.5^0$ and $\theta_C = 105.5^0$

$\Delta E = \int_{\theta B}^{\theta C} (T - \text{Tmean})\, d\theta$

$= \int_{15.5}^{105.5} ((20000 + 9500 \sin 2\theta - 5700 \cos 2\theta) - 20000)\, d\theta$

$\Delta E = 11078$ N-m

$\Delta E = I\, \omega^2\, Cs$

$11078 = I * 18.85^2$

$*0.01\ I = 3121$

Kg.m^2

Angular acceleration of flywheel (α) (θ =45^0)

T excess = T – Tmean

T excess = 9500 sin 2θ – 5700

cos2θ T excess = I x α

9500 sin 2(45) – 5700 cos2(45) = 3121 * α

α = 3.044 rad/s^2

4. The turning moment diagram for a petrol engine is drawn to the following scales: Turning moment, 1mm=5 N-m: crank angle, 1 mm=1°.The turning moment diagram repeats itself at every half revolution of the engine and areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm^2.The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150mm.Determine the coefficient of fluctuation of speed when the engine runs at 1800r.p.m.

Given

m=36kg, k =150mm = 0.15 m, N = 1800 rpm, ω = 188.52 rad/sec

Solution:

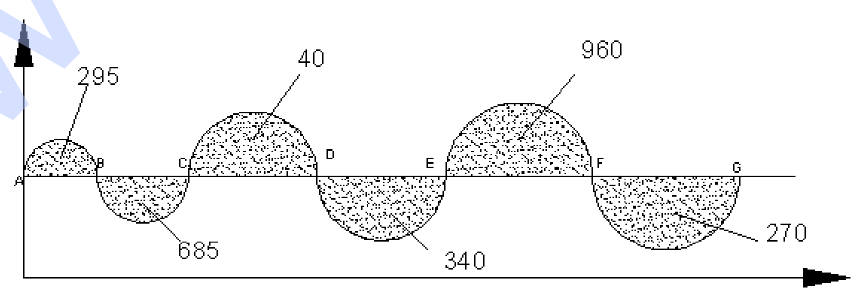
Scale :

1) Turning moment 1mm = 5N-m

2) Crank angle 1mm = 1^0

$= \frac{\pi}{180}\ rad$

Turning moment = 5 xπ/180= 0.08722 N-m



Crank angle θ

A = E

B= E+295

C = E +295 – 685

D = E +295 – 685 + 40

E = E +295 – 685 + 40 – 340

F = E +295 – 685 + 40 – 340 +960

G = E +295 – 685 + 40 – 340 +960 – 270

Maximum energy at B = E+295

Minimum energy at E = E – 690

ΔE = (maximum energy) – (minimum energy)

= (E+295) – (E – 690)[1mm^2 = 0.0872 N-m)

= 985 mm^2

$\Delta E = 985 \times 0.0872 = 85.892$ N-m

$$\Delta E = m k^2 \omega^2 c_s$$

$$85.892 = 36\times0.52^2 \times ((2\pi\times1800)/60) \times c_s$$

$$\mathbf{C_s = 0.003 \text{ (or) } 0.3\%}$$

5.A shaft fitted with a flywheel rotates at 250 r. p .m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution ,the cycle being repeated there after. Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 mm.

Given : N = 250 r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s ; m = 500 kg ; k = 600 mm = 0.6 m

The turning moment diagram for the complete cycle is shown in Fig

We know that the torque required for one complete cycle

= Area of figure OABCDEF

= Area OAEF + Area ABG + Area BCHG + Area CDH

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$

$$= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \frac{1}{2} \times \pi(3000 - 750)$$

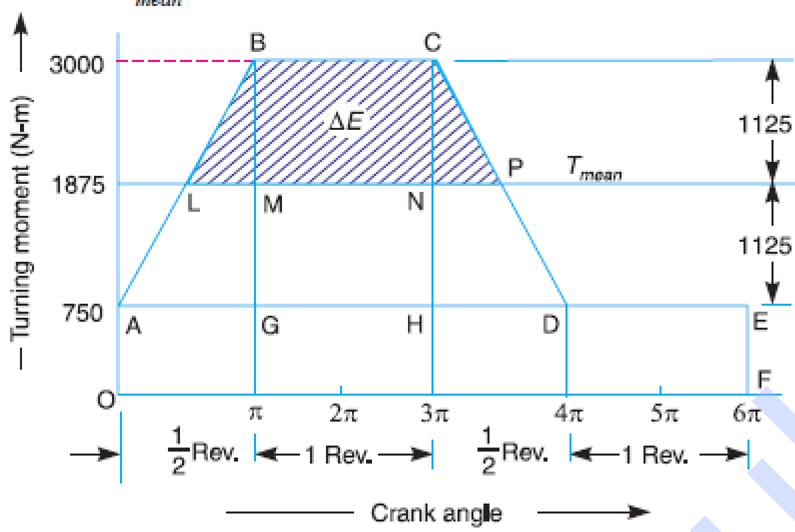
$$= 11\,250\,\pi \text{ N-m} \qquad \ldots(i)$$

If T_{mean} is the mean torque in N-m, then torque required for one complete cycle

$$= T_{mean} \times 6\,\pi \text{ N-m} \qquad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$T_{mean} = 11\,250\,\pi / 6\,\pi = 1875 \text{ N-m}$$



Power required to drive the machine

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 1875 \times 26.2 = 49\,125 \text{ W} = 49.125 \text{ kW} \quad \textbf{Ans.}$$

Coefficient of fluctuation of speed

Let $\qquad C_S$ = Coefficient of fluctuation of speed.

First of all, let us find the values of LM and NP. From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5\,\pi$$

Now, from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5\,\pi$$

From Fig. 16.8, we find that

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore, maximum fluctuation of energy,

$$\Delta E = \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\,\pi \times 1125 + 2\,\pi \times 1125 + \frac{1}{2} \times 0.5\,\pi \times 1125$$

$$= 8837 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.C_S = 500 \times (0.6)^2 \times (26.2)^2 \times C_S = 123\,559\,C_S$$

$$C_S = \frac{8837}{123559} = 0.071 \quad \textbf{Ans.}$$

6.The turning moment diagram for a four stroke gas engine may be assumed for simplicity to be represented by four triangles, the areas of which from the line of zero pressure are as follows : Suction stroke = 0.45 × 10 –3m^2 ; Compression stroke = 1.7 × 10 –3m^2 ; Expansion stroke = 6.8 × 10 –3m^2 ; Exhaust stroke = 0.65 × 10 –3m^2. Each m 2 of area represents 3 MN-m of energy. Assuming the resisting torque to be uniform, find the mass of the rim of a flywheel required to keep the speed between 202 and 198 r.p.m. The mean radius of the rim is 1.2 m.

Solution. Given : $a_1 = 0.45 \times 10^{-3}$ m^2 ; $a_2 = 1.7 \times 10^{-3}$ m^2 ; $a_3 = 6.8 \times 10^{-3}$ m^2; $a_4 = 0.65 \times 10^{-3}$ m^2; $N_1 = 202$ r.p.m; $N_2 = 198$ r.p.m.; $R = 1.2$ m

The turning moment crank angle diagram for a four stroke engine is shown in Fig. 16.12. The areas below the zero line of pressure are taken as negative while the areas above the zero line of pressure are taken as positive.

$\therefore$ Net area $= a_3 - (a_1 + a_2 + a_4)$

$= 6.8 \times 10^{-3} - (0.45 \times 10^{-3} + 1.7 \times 10^{-3} + 0.65 \times 10^{-3}) = 4 \times 10^{-3}$ m^2

Since the energy scale is 1 m^2 = 3 MN-m = 3×10^6 N-m, therefore,

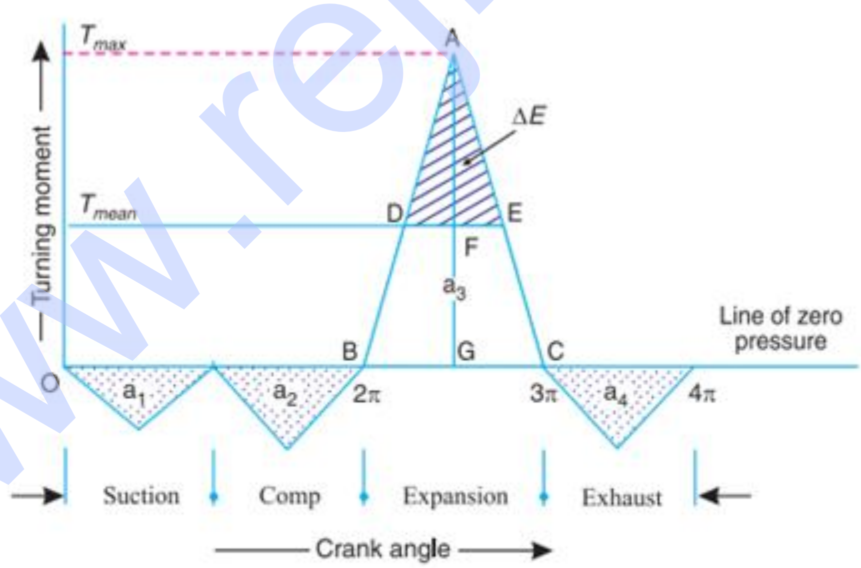
Net work done per cycle $= 4 \times 10^{-3} \times 3 \times 10^6 = 12 \times 10^3$ N-m ... *(i)*

We also know that work done per cycle,

$= T_{mean} \times 4\pi$ N-m ... *(ii)*

From equations *(i)* and *(ii)*,

$T_{mean} = FG = 12 \times 10^3 / 4\pi = 955$ N-m



Work done during expansion stroke

$= a_3 \times$ Energy scale $= 6.8 \times 10^{-3} \times 3 \times 10^6 = 20.4 \times 10^3$ N-m ...*(iii)*

Also, work done during expansion stroke

$$= \text{Area of triangle } ABC$$

$$= \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG = 1.571 \times AG \quad \ldots (iv)$$

From equations ***(iii)*** and ***(iv)***,

$$AG = 20.4 \times 10^3 / 1.571 = 12\,985 \text{ N-m}$$

∴ Excess torque,

$$T_{excess} = AF = AG - FG = 12\,985 - 955 = 12\,030 \text{ N-m}$$

Now from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{12\,030}{12\,985} \times \pi = 2.9 \text{ rad}$$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta ADE = \frac{1}{2} \times DE \times AF = \frac{1}{2} \times 2.9 \times 12030 \text{ N-m}$$

$$= 17\,444 \text{ N-m}$$

Mass of the rim of a flywheel

Let m = Mass of the rim of a flywheel in kg, and

N = Mean speed of the flywheel

$$= \frac{N_1 + N_2}{2} = \frac{202 + 198}{2} = 200 \text{ r.p.m.}$$

We know that the maximum fluctuation of energy (ΔE),

$$17\,444 = \frac{\pi^2}{900} \times m.R^2.N\,(N_1 - N_2) = \frac{\pi^2}{900} \times (1.2)^2\, 200 \times (202 - 198)$$

$$= 12.63\, m$$

∴ $m = 17\,444 / 12.36 = 1381$ kg **Ans.**

UNIT – II BALANCING

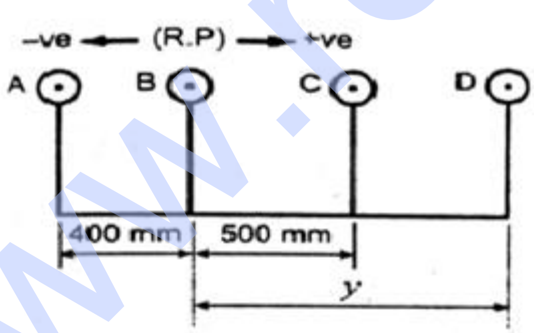
Balancing of rotating masses: The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

Static balancing: The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of the rotation. This is the condition for static balancing.

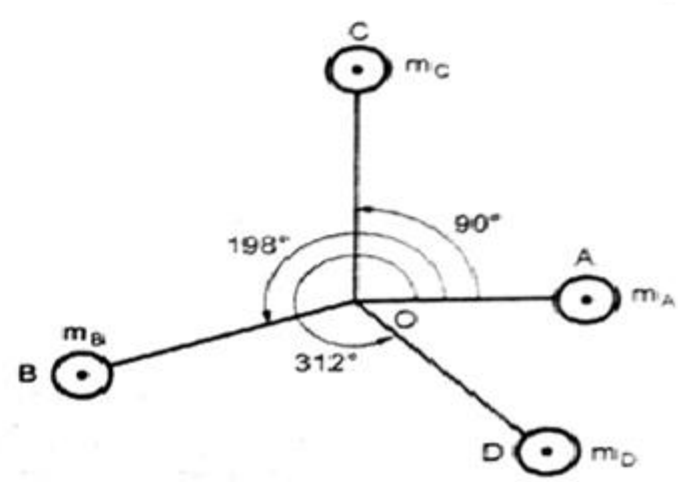
Dynamic balancing: The net couple due to dynamic forces acting on the shaft is equal to zero. The algebraic sum of the moments about any point in the plane must be zero.

1. A rotating shaft carries four masses A,B,C and D which are radially attached to it. The masses centres are 30mm, 38mm, 40mm and 35mm respectively from the axis of rotation. The masses A,C and D are 7.5kg, 5kg, 4kg respectively. The axial distances between the planes of rotation of A and B is 400mm and between B and C is 500mm. The masses A and C are at right angles to each other. Find for a complex (i) the angle between the masses B and D from mass A, (ii) the axial distance between the planes of rotation of C and D, and (iii) the magnitude of mass B.

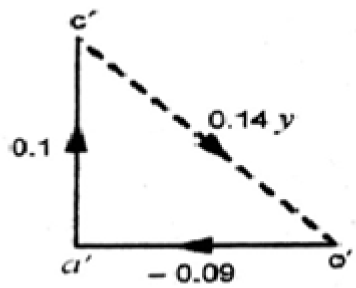
Planes	Mass (m)	Radius r	Centrifugal force $\div\omega^2$	Distance form R.P	Couple $\div\omega^2$
	Kg	m	Kg m	m	Kg m^2
A	7.5	0.03	0.225	-0.4	-0.09
B(RP)	mB	0.038	0.038mB	0	0
C	5	0.04	0.2	0.5	0.1
D	4	0.35	0.14	y	0.14y



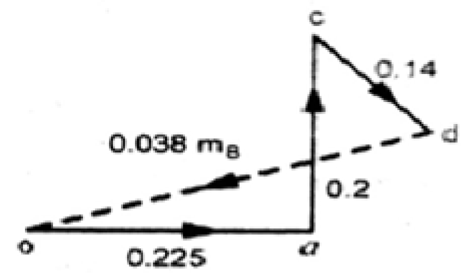
(a) Position of planes



(b) Angular position of planes



(c) Couple polygon



(d) Force polygon

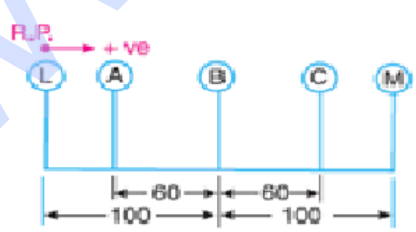
$vector\ b'o' = 0.14y = 0.142kg - m^2$ $\quad y = 1.014m$

$the axial\ distance between\ C and\ D = y - 0.5 = 0.514m$ $\quad vector\ do = 0.038m_B \quad m_B = 9.24kg$

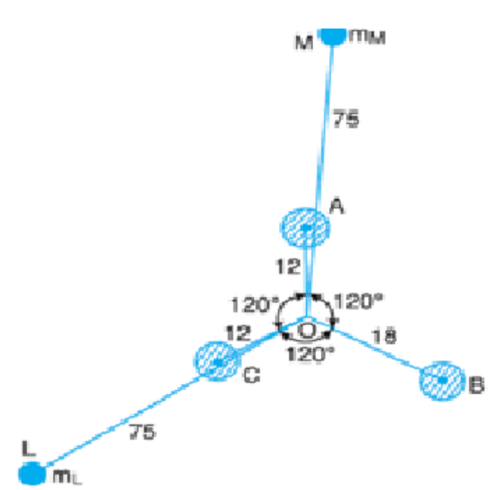
$\theta_B = 198^0$

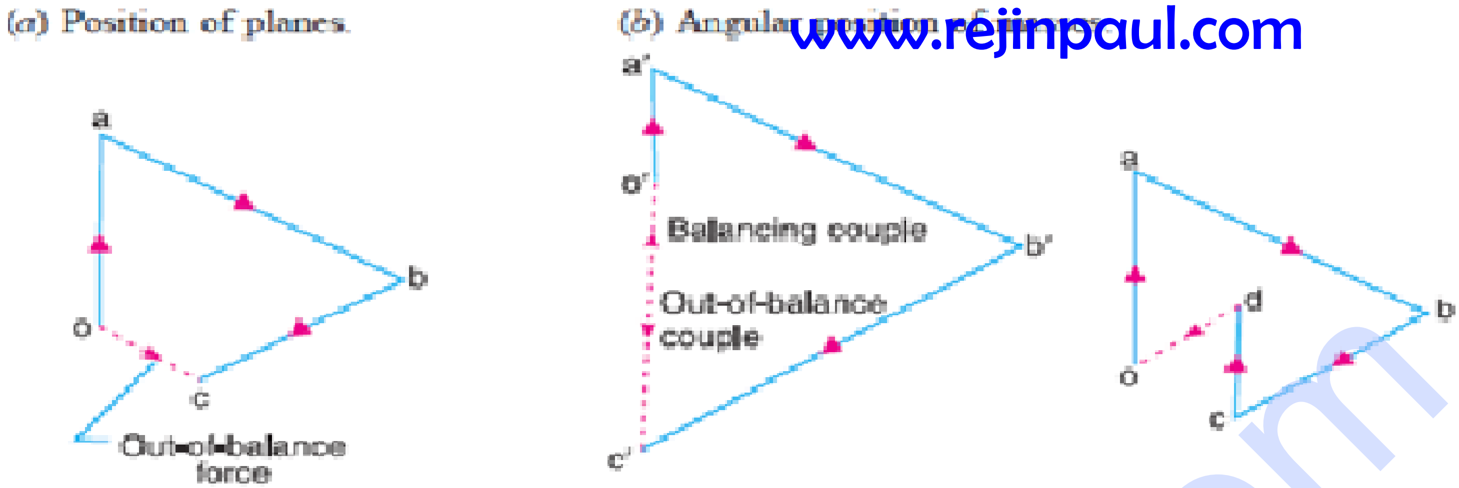
2. A shaft has three eccentrics, each 75mm diameter and 25mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60mm apart. The distance of the centres from the axis of rotation are 12mm, 18mm and 12mm and their angular positions are 120^0 apart. The density of metal is $7000kg/m^3$. Find the amount of out-of-balance force and couple at 600rpm. If the shaft is balanced by adding two masses at a radius 75mm and at distance of 100mm from the central plane of the middle eccentric, find the amount of the masses and their angular positions.

Plane (1)	*Mass* *(m) kg* (2)	*Radius* *(r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from* *plane L.(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
L (R.P.)	m_L	0.075	$75 \times 10^{-3}\ m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75 \times 10^{-3}\ m_M$	0.20	$15 \times 10^{-3}\ m_M$



All dimensions in mm.





Out-of-balance force = vector oc = 4.75 × 10^{-3} kg-m
=4.75 × 10^{-3} × ω^2 = 4.75 × 10^{-3} $(62.84)^2$ = 18.76 N

Out-of-balance couple

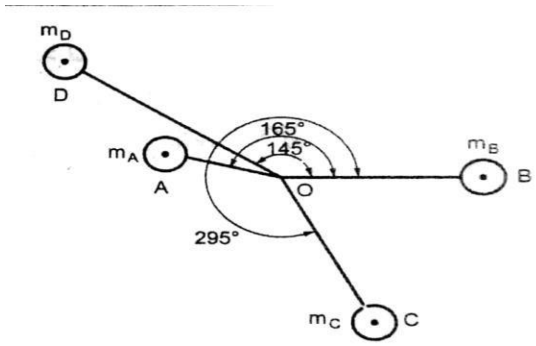
Out-of-balance couple = vector $o'c'$ = 1.1 × 10^{-3} kg-m^2
= 1.1. 10^{-3} × ω^2 = 1.1.$10^{-3}(62.84)^2$ = 4.34 N-m

Amount of balancing masses and their angular positions

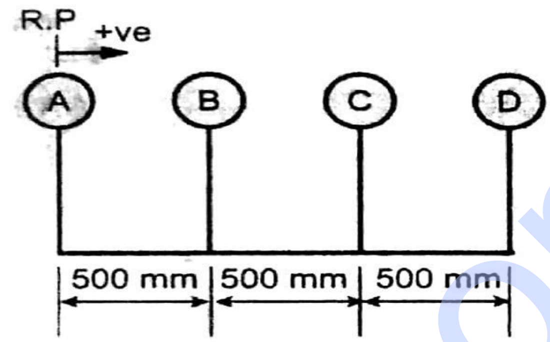
15 × 10^{-3} m_M= vector $c'o'$ = 1.1 × 10^{-3} kg-m^2 Or m_M = 0.073 kg
75 × 10–3 m_L = vector do = 5.2 ×10–3 kg-m
or m_L = 0.0693 kg

3. A,B,C and D are four masses carried by a rotating shaft at radii 100mm, 150mm, 150mm and 200mm respectively. The planes in which the masses rotates are spaced at 500mm apart and the magnitude of the masses B,C and D are 9kg, 5kg and 4kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft must be in complete balance.

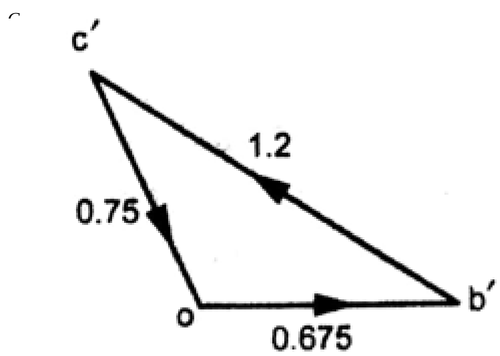
Planes	Mass (m)	Radius r	Centrifugal force ÷ω^2	Distance form R.P	Couple ÷ω^2
	Kg	m	Kg m	m	Kg m^2
A	mA	0.1	0.1 mA	0	0
B(RP)	9	0.15	1.35	0.5	0.675
C	5	0.15	0.75	1	075
D	4	0.2	0.8	1.5	1.2



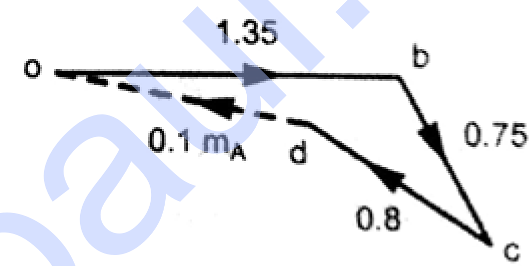
(b) Angular position of planes



(a) Position of planes



(c) Couple polygon



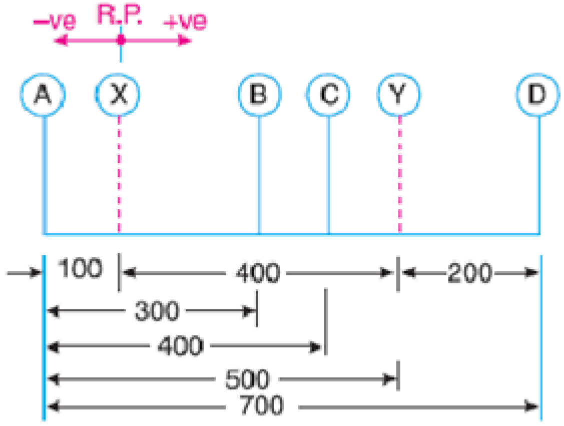
(d) Force polygon

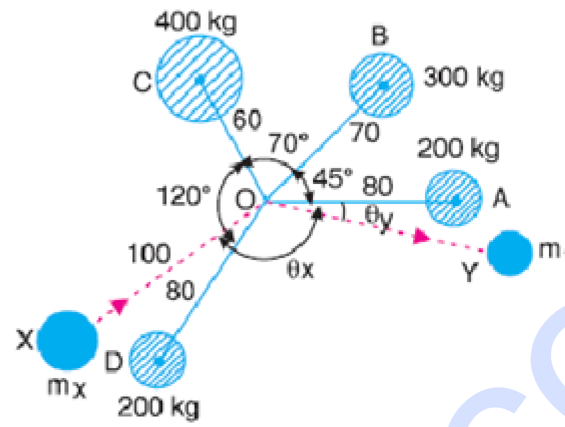
$\theta = 295^0$ $\theta_D = 145^0$ $\theta_A = 165^0$

$Vector\ do = 1.0125 = 0.1 m_A$ $m_A = 10.12 kg$

4.A shaft carries four masses A,B,C and D of magnitude 200kg, 300kg, 400kg and 200kg respectively and revolving at radii 80mm, 70mm 60mm and 80mm in planes measured from A at 300mm,400mm and 700mm. The angles between the cranks measured anticlockwise are A to B 45^0, B to C 70^0 and C to D 120^0. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100mm, between X and Y is 400mm, and between Y and D is 200mm. If the balancing masses revolve at a radius of 100mm, Find their magnitudes and angular positions.

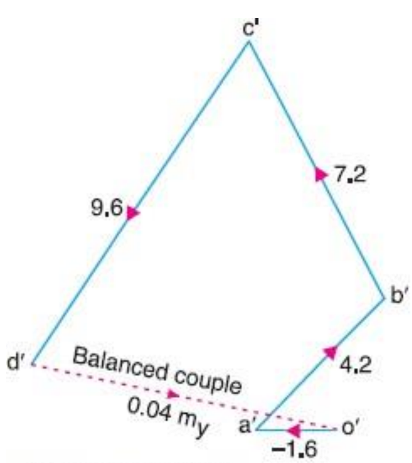
Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from Plane x(l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	200	0.08	16	− 0.1	− 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6



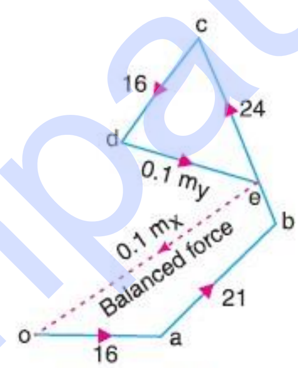


All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.



(*c*) Couple polygon.



(*d*) Force polygon.

$m_Y = 182.5\ kg$ $m_X = 355\ kg$

5. The following data refer to an outside cylinder uncoupled locomotives.

Mass of the reciprocating parts= 300 kg.

Mass of the rotating parts per cylinder = 350kg

Angle between cranks = 90^0

Crank radius = 0.3 Cylinder centers = 1.8m

Radius of balance masses = .08m

Wheel centers =1.5m

If whole of the rotating and 2/3 of the reciprocating parts are to be balanced in planes of the driving wheels,

Find (i) Magnitude and Angular position of balance masses.

Speed in km/hr at which the wheel lift off the rails when the load on each driving wheels is 30KN and the diameter of tread driving wheels is 1.8m and Swaying couple at speed found in second plane

Solution : Given : $m_1 = 360$ kg ; $m_2 = 300$ kg ; $\angle AOD = 90°$; $r_A = r_D = 0.3$ m ; $a = 1.75$ m ; $r_B = r_C = 0.75$ m ; $c = 2/3$.

We know that the equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300 = 560 \text{ kg}$$

1. *Magnitude and angular position of balance masses*

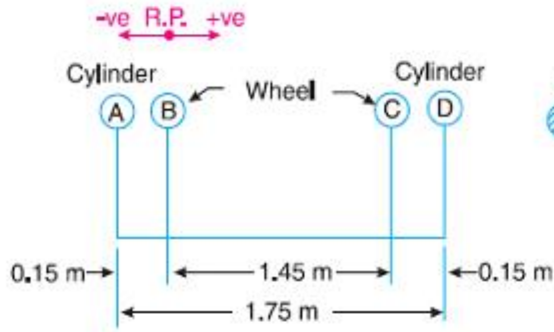
Let m_B and m_C = Magnitude of the balance masses, and

θ_B and θ_C = angular position of the balance masses m_B and m_C from the crank A.

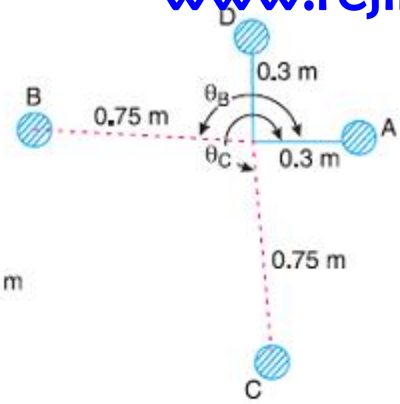
The magnitude and direction of the balance masses may be determined, graphically, as discussed below :

1. First of all, draw the positions of the planes of the wheels and the cylinders as shown in Fig. 22.11 (*a*). Since the cranks of the two cylinders are at right angles, therefore assuming the position of the cylinder A in the horizontal direction, draw OA and OD at right angles to each other as shown in Fig. 22.11 (*b*).
2. Assuming the plane of wheel B as the reference plane, the data may be tabulated as below:

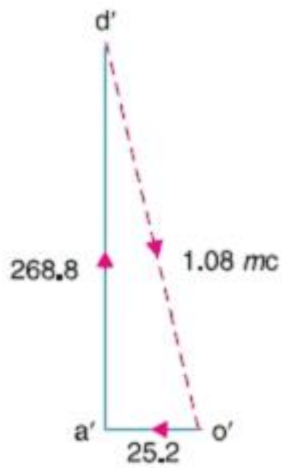
Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane L (l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m²* (6)
L (R.P.)	m_L	0.075	$75 \times 10^{-3}\ m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75 \times 10^{-3}\ m_M$	0.20	$15 \times 10^{-3}\ m_M$



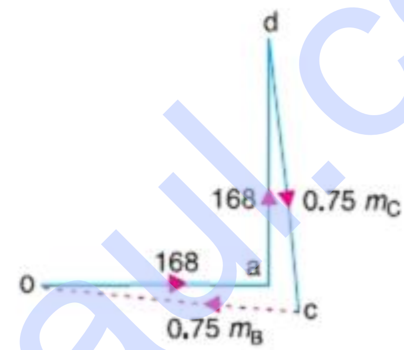
(a) Position of planes.



(b) Position of masses.



(c) Couple polygon.



(d) Force polygon.

$\theta_C = 275°$ **Ans.** $m_B = 249$ kg **Ans.** $\theta_B = 174.5°$ **Ans.**

Speed at which the wheel will lift off the rails:

Given : $P = 30$ kN $= 30 \times 10^3$ N ; $D = 1.8$ m

Let ω = Angular speed at which the wheels will lift off the rails in rad/s, and

v = Corresponding linear speed in km/h.

We know that each balancing mass,

$$m_B = m_C = 249 \text{ kg}$$

$\therefore$ Balancing mass for reciprocating parts,

$$B = \frac{c.m_2}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249 = 89 \text{ kg}$$

We know that $\omega = \sqrt{\frac{P}{B.b}} = \sqrt{\frac{30 \times 10^3}{89 \times 0.75}} = 21.2$ rad/s ...($\because b = r_B = r_C$)

$$v = \omega \times D/2 = 21.2 \times 1.8/2 = 19.08 \text{ m/s}$$

$$= 19.08 \times 3600/1000 = 68.7 \text{ km/h}$$ **Ans.**

Swaying couple at the speed

We know that the swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2 . \omega^2 . r = \frac{1.75\left[1-\frac{2}{3}\right]}{\sqrt{2}} \times 300(21.2)^2 0.3 \text{ N-m}$$

$= 16\ 687$ N-m $= 16.687$ kN-m **Ans.**

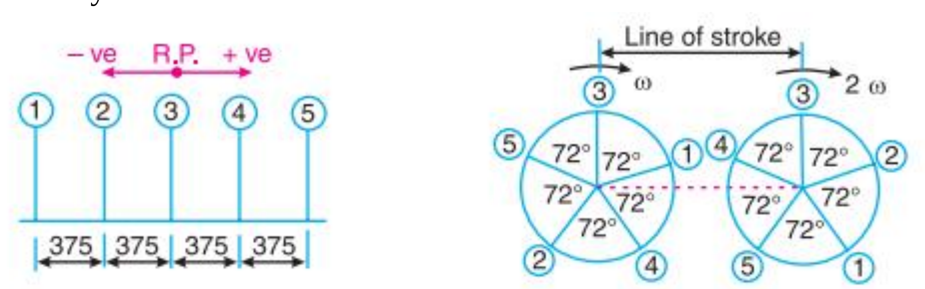
6. A five cylinder in-line engine running at 750 r.p.m. has successive cranks 144° apart, the distance between the cylinder centre lines being 375 mm. The piston stroke is 225mm and the ratio of the connecting rod to the crank is 4. Examine the engine for balance of primary and secondary forces and couples. Find the maximum values of these and the position of the central crank at which these maximum values occur. The reciprocating mass for each cylinder is 15 kg.

Solution.Given : N= 750 r.p.m. or 2 ω= π× 750/60 = 78.55 rad/s ; L= 225 mm = 0.225 m or r = 0.1125 m ; n= l/r= 4 ; m = 15 kg

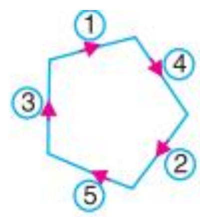
Assuming the engine to be a vertical engine, the positions of the cylinders and the cranks are shown in Fig. (a), (b) and (c). The plane 3 may be taken as the reference plane and the crank 3 as the reference crank. The data may be tabulated as given in the following table.

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from ref. Plane 3 (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
1	15	0.1125	1.6875	– 0.75	– 1.265
2	15	0.1125	1.6875	– 0.375	– 0.6328
3(*R.P.*)	15	0.1125	1.6875	0	0
4	15	0.1125	1.6875	+ 0.375	+ 0.6328
5	15	0.1125	1.6875	+ 0.75	+ 1.265

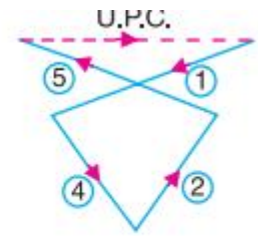
Now, draw the force and couple polygons for primary and secondary cranks as shown in Fig. (d), (e), (f), and (g). Since the primary and secondary force polygons are close, therefore the engine is balanced for primary and secondary forces. **Ans**



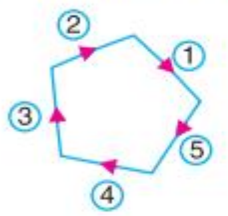
(*a*) Position of planes. (*b*) Primary crank positions. (*c*) Secondary crank positions.



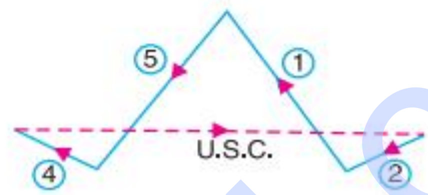
(*d*) Primary force polygon.



(*e*) Primary couple polygon.



(*f*) Secondary force polygon.



(*g*) Secondary couple polygon.

Maximum unbalanced primary couple We know that the closing side of the primary couple polygon [shown dotted in Fig.(e)] gives the maximum unbalanced primary couple. By measurement, we find that maximum unbalanced primary couple is proportional to 1.62 kg-m^2

. ∴Maximum unbalanced primary couple,

$$U.P.C. = 1.62 \times \omega^2 = 1.62\ (78.55)^2 = 9996 \text{ N-m } \textbf{Ans.}$$

We see from (e) [shown by dotted line] that the maximum unbalanced primary couple occurs when crank 3 is at 90° from the line of stroke. Maximum unbalanced secondary couple

We know that the closing side of the secondary couple polygon [shown dotted in Fig.(g)] gives the maximum unbalanced secondary couple. By measurement, we find that maximum unbalanced secondary couple is proportional to 2.7 kg-m^2

.

∴Maximum unbalanced secondary couple.

$$U.S.C = 2.7 \times \frac{\omega^2}{n} = 2.7 \times \frac{(78.55)^2}{4} = 4165 \text{ N-m } \textbf{Ans.}$$

UNIT –III FREE VIBRATIONS

Types of vibration:

a) The actuating force on the body
b) The stresses in the supporting medium
   1. According to the Actuating force:
      (a) Free or Natural vibrations
      (b) Forced vibrations
      (c) Damped vibrations
      (d) Undamped vibrations
   2. According to motion of system with respect to axis
      (a) Longitudinal vibrations
      (b) Transverse vibrations
      (c) Torsional vibrations

Terms used vibratory motion: (a)Time period (or)period of vibration: It is the time taken by a vibrating body to repeat the motion itself. time period is usually expressed in seconds.

(b) Cycle: It is the motion completed in one time period.

(c) Periodic motion: A motion which repeats itself after equal interval of time.

(d)Amplitude (X) The maximum displacement of a vibrating body from the mean position.it is usually expressed in millimeter.

(e) Frequency (f) The number of cycles completed in one second is called frequency

Degrees of freedom: The minimum number of independent coordinates required to specify the motion of a system at any instant is known as D.O.F of the system.

1. A vibrating system consists of a mass of 8kg spring of stiffness 5.6N/mm and a dashpot of damping coefficient of 40N/m/s. Find
   (a) the critical damping coefficient
   (b) the damping factor
   (c) the natural frequency of damped vibration,
   (d) the logarithmic decrement,
   (e) the ratio of two consecutive amplitudes, and
   (f) the number of cycles after which the original amplitude is reduced to 20 percent

Given Data: $m = 8$ kg; $s = 5.6$ N/mm $= 5.6 \times 10^3$ N/m; $C = 40$ N/m/s

Solution:

(a) Critical damping coefficient (c_c):

We know that $$c_c = 2\,m\,\omega_n = 2\,m\sqrt{\frac{s}{m}} = 2\sqrt{s \cdot m}$$

$$\therefore \quad c_c = 2\sqrt{5.6 \times 10^3 \times 8} = \mathbf{422.32\ N/m/s}$$ **Ans.**

(b) Damping factor (ζ):

$$\text{Damping factor, } \zeta = \frac{c}{c_c} = \frac{40}{422.32} = \mathbf{0.0945} \quad \textbf{Ans.}$$

(c) Natural frequency of damped vibration (f_d):

We know that the circular frequency of damped vibrations,

$$\omega_d = \sqrt{1-\zeta^2} \cdot \omega_n$$

where $$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{5.6 \times 10^3}{8}} = 26.34 \text{ rad/s}$$

$\therefore$ $$\omega_d = \sqrt{1-(0.0945)^2} \times 26.34 = 26.22 \text{ rad/s}$$

$\therefore$ Natural frequency of damped vibration of the system

$$f_d = \frac{\omega_d}{2\pi} = \frac{26.22}{2\pi} = \mathbf{4.173\ Hz} \quad \textbf{Ans.}$$

(d) Logarithmic decrement (δ):

We know that logarithmic decrement,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.0945)}{\sqrt{1-(0.0945)^2}} = \mathbf{0.596} \quad \textbf{Ans}$$

(e) Ratio of two consecutive amplitudes $\left(\dfrac{X_n}{X_{n+1}}\right)$:

Let X_n and X_{n+1} = Magnitudes of two consecutive amplitudes

The logarithmic decrement can be given by

$$\delta = ln\left[\frac{X_n}{X_{n+1}}\right] \quad \text{or} \quad \frac{X_n}{X_{n+1}} = e^{\delta}$$

$\therefore$ $$\frac{X_n}{X_{n+1}} = e^{0.596} = \mathbf{1.8156} \quad \textbf{Ans.}$$

(f) Number of cycles after which the amplitude is reduced to 20% (n):

Let X_0 = Amplitude at the starting position,

X_n = Amplitude after n cycle = 20% X_0 = 0.2 X_0

The logarithmic decrement in terms of number of cycles (n) is given by

$$\delta = \frac{1}{n} \cdot \ln\left(\frac{X_0}{X_n}\right)$$

or
$$0.596 = \frac{1}{n} \ln\left(\frac{X_0}{0.2\,X_0}\right)$$

or
$$n = 2.7 \text{ cycles} \quad \textbf{Ans.}$$

2. Derive the expression for the natural frequency of free transverse or longitud inal vibrations by using any two methods.

<u>Equilibrium Method</u>

Consider a constraint (i.e. spring) of negligible mass in an unstrained position, as shown in Figure

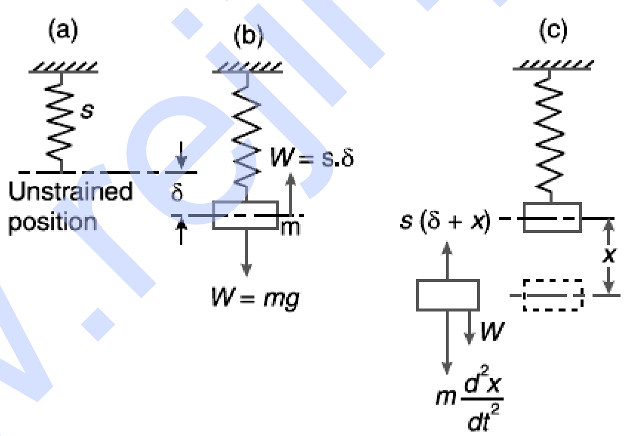
Let s = Stiffness of the constraint. It is the force required to produce unit d isplacement in the direction of vibration. It is usually expressed in N/m.

m = Mass of the body suspended from the constraint in kg

W = Weight of the body in newtons = m.g,

δ= Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



Restoring force =W - s (δ + x) =W - s.δ - s.x

= s.δ - s.δ - s. x = -s.x (since W = s.δ). . (i)

(Taking upward force as negative)

And Accelerating force = Mass × Acceleration = m x .... (ii)$\frac{d^2x}{dt^2}$

(Taking downward force as +ve)

Equating Restoring force and accelerating force

m x = -s.x$\frac{d^2x}{dt^2}$

$$m\frac{d^2x}{dt^2} + s.x = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}.x = 0 \ldots\ldots\ldots\ldots (iii)$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \ldots\ldots.. (iv)$$

$$\omega = \sqrt{\frac{s}{m}}$$

Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$

Natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

ii) Energy Method:

In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero. In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times.

$$\frac{d}{dt}(K.E + P.E) = 0$$

$$\frac{1}{2}m\left[\frac{dx}{dt}\right]^2$$

$$\left[\frac{0+S.X}{2}\right] \quad \frac{1}{2}sx^2$$

$$\frac{d}{dt}\left(\frac{1}{2}m\left[\frac{dx}{dt}\right]^2 + \frac{1}{2}sx^2\right)$$

$$\frac{1}{2}m2\left(\frac{dx}{dt}\right)\left(\frac{d^2x}{dt^2}\right) \quad \frac{1}{2}s2x\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} + sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0$$

The time period and the natural frequency may be obtained as discussed in the previous method.

3. A steel shaft 1.5 m long is 95 mm in diameter for the first 0.6 m of its length, 60 mm in diameter for the next 0.5 m of the length and 50 mm in diameter for the remaining 0.4 m of its length. The shaft carries two flywheels at two ends, the first having a mass of 900 kg and 0.85 m radius of gyration located at the 95 mm diameter end and the second having a mass of 700 kg and 0.55 m radius of gyration located at the other end. Determine the location of the node and the natural frequency of free torsional vibration of the system. The modulus of rigidity of shaft material may be taken as 80 GN/m2.

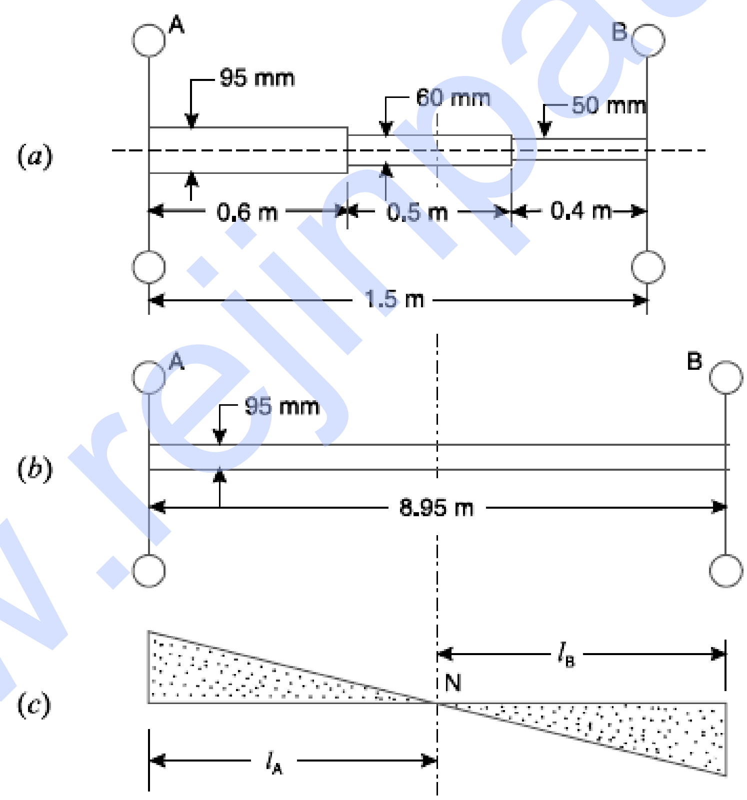
Given:

$L = 1.5$ m, $d_1 = 95$mm, $l_1 = 0.6$ m, $d_2 = 60$mm, $l_2 = 0.5$m, $d_3 = 50$ mm, $l_3 = 0.4$m,

$m_A = 900$kg, $K_A = 0.85$m, $m_B = 700$kg, $K_B = 0.85$m, $C = 80 \times 10^9$ N/m^2

Solution:

Equivalent shaft Diagram:



$$l = l_1 + l_2 \left(\frac{d1}{d2}\right)^4 + l_3\left(\frac{d1}{d3}\right)^4$$

$l = 0.6+0.5(0.095/0.060)+ 0.4(0.095/0.050) = 8.95$ m

b. Location of the node

$$I_A = m_A K_A{}^2 = 900\ (0.85)^2 = 650\text{kg/m}^2$$

$$I_B = m_B K_B{}^2 = 700\ (0.85)^2 = 212\ \text{kg/m}^2$$

$$I_A l_A = I_B l_B$$

$$l_A = \frac{I_B}{I_A} l_B = \frac{212}{650} l_B = 0.326 l_B$$

Finding l_B

Also, $l_A + l_B = l = 8.95\,m$

$$0.326\,l_B + l_B = 8.95$$
$$1.326\,l_B = 8.95$$

$$\boldsymbol{l_B = 6.75\,m}$$

Finding l_A

$$l_A + l_B = 8.95$$

$$l_A + 6.75 = 8.95$$

$$\boldsymbol{l_A = 2.20m}$$

Hence the node lies at 2.2m from flywheel A on the equivalent shaft.

c. Position of the node on the original shaft:

$$l_A + (l_A - l_1)\left(\frac{d_2}{d_1}\right)$$

$$2.2 + (2.2 - 0.6)\left(\frac{0.06}{0.095}\right)$$

Position of the node on the shaft 0.855

Natural frequency of free torsional vibrations J=0.095^4 = 8 x10^{-6}m^4

$$\boldsymbol{f_{nA} = f_{nB}}$$

Natural frequency = 3.37Hz

4. A machine of mass 75kgis mounted on springs and is fitted with a dashpot to damp out vibrations. There are three springs each of stiffness 10N/mm and it is found that the amplitude of vibrations diminishes from 38.4mm to 6.4mm in two complete oscillations. Assuming that the damping force varies as the velocity, determine

i. The resistance of the dashpot at unit velocity.

ii. The ratio of frequency of the damped vibrations to the frequency of undamped vibrations and iii.The periodic time of the damped vibrations.

Given:

m = 75 kg;
s = 10 N/mm = 10 ×10^3 N/m ; x_1 = 38.4 mm = 0.0384 m ;
x_3 = 6.4 mm =0.0064 m

Solution:

Since the stiffness of each spring is 10 × 103 N/m and there are 3 springs, therefore total stiffness,

$$s = 3\times10\times10^3 = 30\times10^3 \text{ N/m}$$

Natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{30\ X\ 10^3}{75}} = 20\ rad/s$$

a. Resistance of the dashpot at unit velocity

Let

x_2 = Amplitude after one complete oscillation in metres, and

x_3 = Amplitude after two complete oscillations in metres.

Solution:

Since the stiffness of each spring is 10 × 103 N/m and there are 3 springs, therefore total stiffness,

$$s = 3\times10\times10^3 = 30\times10^3 \text{ N/m}$$

Natural circular frequency of motion

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{30\ X\ 10^3}{75}} = 20\ rad/s$$

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/2} = \left(\frac{0.0384}{0.0064}\right)^{1/2} = 2.45$$

$$\frac{x_1}{x_2} = \frac{x_2}{x_3}$$

$$\left(\frac{x_1}{x_2}\right)^2 = \frac{x_1}{x_3}$$

$$\left[\frac{x_1}{x_3}\right] = \frac{x_1}{x_2}\left(\frac{x_2}{x_3}\right) = \frac{x_1}{x_2}\left(\frac{x_1}{x_2}\right) = \left(\frac{x_1}{x_2}\right)^2$$

c = Resistance of the dashpot in newtons at unit velocity i.e. in N/m/s

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{2\pi a}{\sqrt{{\omega_n}^2 - a^2}}$$

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{2\pi a}{\sqrt{20^2 - a^2}}$$

,

$$\log_e 2.45 = \frac{2\pi a}{\sqrt{20^2 - a^2}}$$

$$0.8951 = \frac{2\pi a}{\sqrt{20^2 - a^2}}$$

Squaring on both sides:

$$0.8 = \frac{39.5a^2}{400^2 - a^2}$$

$a^2 = 7.84$

a=2.8

$$\log_e 2.45 = a \times \frac{2\pi}{\sqrt{(20)^2 - a^2}}$$

$$0.8951 = \frac{a \times 2\pi}{\sqrt{400 - a^2}} \quad \text{or} \quad 0.8 = \frac{a^2 \times 39.5}{400 - a^2} \quad \text{. . . (Squaring both sides)}$$

∴ $a^2 = 7.94$ or $a = 2.8$

We know that $a = c / 2m$

∴ $c = a \times 2m = 2.8 \times 2 \times 75 = 420$ N/m/s

2. Ratio of the frequency of he damped vibration to the frequency of undamped vibration

$$f_{n1} = \text{Frequency of damped vibration} = \frac{\omega_d}{2\pi}$$

$$f_{n2} = \text{Frequency of undamped vibration} = \frac{\omega_n}{2\pi}$$

$$\frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{(\omega_n)^2 - a^2}}{\omega_n} = \frac{\sqrt{(20)^2 - (2.8)^2}}{20} = 0.99$$

3.Periodic time of damped vibration:

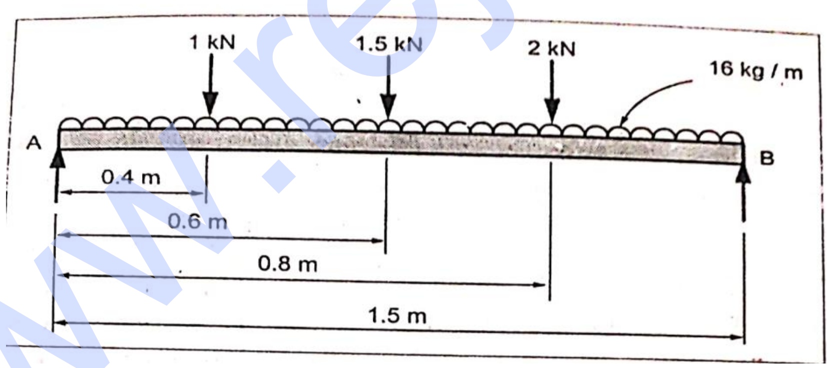
We know that periodic time of damped vibration

$$= \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{2\pi}{\sqrt{(20)^2 - (2.8)^2}} = 0.32 \text{ s}$$

5.A shaft 30mm diameter and 1.5 long has a mass of 16kg/m length. It is simply supported at the ends and carries isolated loads 1kN, 1.5kN and 2kN at 0.4m, 0.6m and 0.8m respectively from the left support. Find the frequency of the transverse vibrations: 1. Neglecting the mass of the shaft, and 2. Considering the mass of the shaft. Take E= 200GPa.

Given : d =30mm = 0.03m, l = 1.5m, m = 16kg/m, E = 200GPa = 200 x 10^9N/m^2

Solution:



Moment of inertia of the shaft, $I = \frac{\pi}{64} d^4 = \frac{\pi}{64} (0.03)^4 = 3.976 \times 10^{-8} \text{ m}^4$

Static deflection due to a load of 1 kN,

$$\delta_1 = \frac{W a^2 b^2}{3 \text{ EI} \cdot l} = \frac{1000 (0.4)^2 (1.1)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5} = 5.41 \times 10^{-3} \text{ m}$$

... [Here $a = 0.4$ m, and $b = 1.1$ m]

Similarly, static deflection due to a load of 1.5 kN,

$$\delta_2 = \frac{W\,a^2\,b^2}{3\,EI \cdot l} = \frac{1500\,(0.6)^2\,(0.9)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5} = 0.01222 \text{ m}$$

... [Here $a = 0.6$ m, and $b = 0.9$ m]

Static deflection due to a load of 2 kN,

$$\delta_3 = \frac{W\,a^2\,b^2}{3\,EI \cdot l} = \frac{2000\,(0.8)^2\,(0.7)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5} = 0.01752 \text{ m}$$

... [Here $a = 0.8$ m, and $b = 0.7$ m]

and static deflection due to the mass of the shaft (*i.e.*, a *udl*),

$$\delta s = \frac{5}{384} \times \frac{w\,l^4}{EI} = \frac{5}{384} \times \frac{(16 \times 9.81)\,(1.5)^4}{200 \times 10^9 \times 3.976 \times 10^{-8}} = 1.301 \times 10^{-3} \text{ m}$$

(a) Neglecting the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{5.41 \times 10^{-3} + 0.01222 + 0.01752}} = \textbf{2.659 Hz} \quad \textbf{Ans.}$$

(b) Considering the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \dfrac{\delta_s}{1.27}}} = \frac{0.4985}{\sqrt{5.41 \times 10^{-3} + 0.01222 + 0.01752 + \left(\dfrac{1.301 \times 10^{-3}}{1.27}\right)}}$$

$$= \textbf{2.62 Hz} \quad \textbf{Ans.}$$

6.A shaft 1.5m long, supported in flexible bearing at the ends, carries two wheels each of 60kg mass. One wheel is situated at the centre of the shaft and the other at a distance of 375mm form the centre. The shaft is hollow of external diameter 75mm and inner diameter 40mm. The density of the shaft material is 7700kg/m^3. Find the frequency of transverse vibration. Take E = 200GPa.

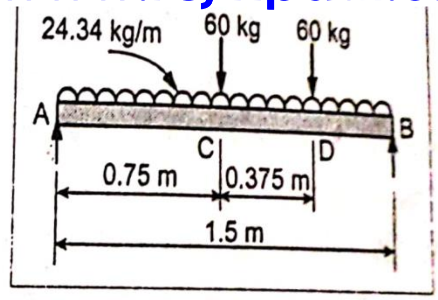
Given : l =1.5m,

Do =75mm = 0.075mm,

Di =40mm = 0.040mm ,

ρ = 7700 kg/m^3

E = 200GPa = 200x10^{9}N/m^2



☺ *Solution:* A shaft supported in flexible bearings is assumed to be a simply supported beam. The given shaft is shown in Fig.8.10.

Since the density of shaft material is given as 7700 kg/m^3, therefore mass of the shaft per metre length

$$m_s = \text{Area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}[0.075^2 - 0.04^2] \times 1 \times 7700 = 24.34 \text{ kg/m}$$

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64}[D_0^4 - D_i^4] = \frac{\pi}{64}[(0.075)^4 - (0.04)^4] = 1.427 \times 10^{-6} \text{ m}^4$$

We know that static deflection due to a mass of 60 kg at C,

$$\delta_1 = \frac{W_1 a^2 b^2}{3\,EI \cdot l} = \frac{(60 \times 9.81)(0.75)^2(0.75)^2}{3(200 \times 10^9)(1.427 \times 10^{-6})\,1.5} = 1.45 \times 10^{-4} \text{ m}$$

... [Here $a = 0.75$ m, and $b = 0.75$ m]

Similarly, static deflection due to a mass of 60 kg at D,

$$\delta_2 = \frac{W_2 a^2 b^2}{3\,EI \cdot l} = \frac{(60 \times 9.81)(1.125)^2(0.375)^2}{3(200 \times 10^9)(1.427 \times 10^{-6})\,1.5} = 8.157 \times 10^{-5} \text{ m}$$

... [Here $a = 0.75 + 0.375 = 1.125$ m, and $b = 1.5 - 1.125 = 0.375$ m]

We know that static deflection due to mass of the shaft,

$$\delta_s = \frac{5}{384} \times \frac{w\,l^4}{EI} = \frac{5}{384} \times \frac{(23.34 \times 9.81)(1.5)^4}{(200 \times 10^9) \times (1.427 \times 10^{-6})} = 5.288 \times 10^{-5} \text{ m}$$

... [Here $w = m_s \times g$]

Therefore the frequency of transverse vibration, according to Dunkerley's equation, is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \dfrac{\delta_s}{1.27}}}$$

$$= \frac{0.4985}{\sqrt{(1.45 \times 10^{-4}) + (8.157 \times 10^{-5}) + \left(\dfrac{5.288 \times 10^{-5}}{1.27}\right)}} = \textbf{30.44 Hz} \quad \textbf{Ans.}$$

UNIT-IV FORCED VIBRATION

Forced Vibration:

When the body vibrates under the influence of external force, then the body is said to be under forced vibration.

Examples of forced vibration:

1. Ringing of electric bell.
2. Vibration of various machines like air compressor, IC engines, Machine tools and mobile cranes.

Types of external Excitation:

1. Periodic forces,
2. Impulse type of forces,
3. Random Forces.

Periodic forces are further classified into harmonic and non-harmonic forces. Vibration because of impulsive forces is called as transient. Earthquake and acoustic excitation are typical examples of random forces. In this chapter we would be analyzing only about periodic forcing functions.

1.A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine has a mass of 20 kg and move through a vertical stroke of 150 mm with simple harmonic motion. A dashpot is provided whose damping resistance is directly proportional to the velocity and amounts to 1.5 kN per metre per second. Considering that the steady state of vibration is reached; determine: 1. the amplitude of forced vibrations, when the driving shaft of the engine rotates at 480 r.p.m., and 2. the speed of the driving shaft at which resonance will occur.

Solution : Given. m = 300 kg; δ = 2 mm = 2 × 10–3 m ; $m1$ = 20 kg ; l = 150 mm = 0.15 m ; c = 1.5 kN/m/s = 1500 N/m/s ; N = 480 r.p.m. or ω = 2π×480 / 60 = 50.3 rad/s

Amplitute of forced vibration:

We know that stiffness of the frame,

$$s = m.g / \delta = 300 \times 9.81/2 \times 10^{-3} = 1.47 \times 10^{6} \text{ N/m}$$

Since the length of stroke (l) = 150 mm = 0.15 m, therefore radius of crank,

$$r = l / 2 = 0.15 / 2 = 0.075 \text{ m}$$

We know that the centrifugal force due to the reciprocating parts or the static force,

$$F = m_1.\omega^2.r = 20\,(50.3)^2\,0.075 = 3795 \text{ N}$$

∴ Amplitude of the forced vibration (maximum),

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{3795}{\sqrt{(1500)^2(50.3)^2 + [1.47\times10^6 - 300(50.3)^2]^2}}$$

$$= \frac{3795}{\sqrt{5.7\times10^9 + 500\times10^9}} = \frac{3795}{710\times10^3} = 5.3\times10^{-3} \text{ m}$$

$= 5.3$ mm **Ans.**

2.Speed of driving shaft at which resonance occur.

Let N = Speed of the driving shaft at which the resonance occurs in r.p.m.

We know that the angular speed at which the resonance occurs,

$$\omega = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.47\times10^6}{300}} = 70 \text{ rad/s}$$

$\therefore$ $N = \omega\times60/2\pi = 70\times60/2\pi = 668.4$ r.p.m. **Ans.**

2.A machine part of mass 2 kg vibrates in a viscous medium. Determine the damping coefficient when a harmonic exciting force of 25 n results in resonant amplitude of 12.5 mm with a period of 0.2 second. if the system is excited by a harmonic force of frequency 4 hz what will be the percentage increase in the amplitude of vibration when damper is removed as compared with that with damping.

Given :
m = 2 kg ; F = 25 N ; Resonant $xmax$ = 12.5 mm = 0.012 5 m tp = 0.2 sec, f = 4Hz

Solution:

Damping coefficient :

Let c = Damping coefficient in N/m/s.

We know that natural circular frequency of the exicting force,

$$\omega_n = 2\pi/t_p = 2\pi/0.2 = 31.42 \text{ rad/s}$$

We also know that the maximum amplitude of vibration at resonance (x_{max}).

$$0.0125 = \frac{F}{c.\omega_n} = \frac{25}{c\times31.42} = \frac{0.796}{c} \text{ or } c = 63.7 \text{ N/m/s}$$ **Ans.**

Percentage increase in amplitude:

Since the system is excited by a harmonic force of frequency (f) = 4 Hz, therefore corresponding circular frequency

$$\omega = 2\pi \times f = 2\pi \times 4 = 25.14 \text{ rad/s}$$

We know that maximum amplitude of vibration with damping,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{25}{\sqrt{(63.7)^2(25.14)^2 + [2(31.42)^2 - 2(25.14)^2]^2}}$$

$$\ldots \left[\because (\omega_n)^2 = s/m \quad \text{or} \quad s = m(\omega_n)^2\right]$$

$$= \frac{25}{\sqrt{2.56\times10^6 + 0.5\times10^6}} = \frac{25}{1749} = 0.0143 \text{ m} = 14.3 \text{ mm}$$

and the maximum amplitude of vibration when damper is removed,

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - \omega^2\right]} = \frac{25}{2[(31.42)^2 - (25.14)^2]} = \frac{25}{710} = 0.0352 \text{ m}$$

$$= 35.2 \text{ mm}$$

$\therefore$ Percentage increase in amplitude

$$= \frac{35.2 - 14.3}{14.3} = 1.46 \quad \text{or} \quad 146\% \text{ Ans.}$$

3.A machine has a mass of 100 kg and unbalanced reciprocating parts of mass 2 kg which move through a vertical stroke of 80 mm with simple harmonic motion. The machine is mounted on four springs, symmetrically arranged with respect to centre of mass, in such a way that the machine has one degree of freedom and can undergo vertical displacements only. Neglecting damping, calculate the combined stiffness of the spring in order that the force transmitted to the foundation is 1 / 25 th of the applied force, when the speed of rotation of machine crank shaft is 1000 r.p.m. When the machine is actually supported on the springs, it is found that the damping reduces the amplitude of successive free vibrations by 25%. Find: 1. the force transmitted to foundation at 1000 r.p.m., 2. the force transmitted to the foundation at resonance, and 3. the amplitude of the forced vibration of the machine at resonance.

Given : $m1$ = 100 kg ; $m2$ = 2 kg ; l = 80 mm = 0.08 m ; ε = 1 / 25 ;
N = 1000 r.p.m. or ω = 2π×1000 / 60 = 104.7 rad/s

Solution:

Combined stiffness of the spring:

Let s = Combined stiffness of springs in N/m, and

ω_n = Natural circular frequency of vibration of the machine in rad/s.

We know that transmissibility ratio (ε),

$$\frac{1}{25} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(104.7)^2 - (\omega_n)^2}$$

$$(104.7)^2 - (\omega_n)^2 = 25(\omega_n)^2 \quad \text{or} \quad (\omega_n)^2 = 421.6 \text{ or } \omega_n = 20.5 \text{ rad/s}$$

We know that $\omega_n = \sqrt{s/m_1}$

$\therefore$ $s = m_1 (\omega_n)^2 = 100 \times 421.6 = 42\ 160$ N/m **Ans.**

Force transmitted to the foundation at 1000rpm :

Let F_T = Force transmitted, and

x_1 = Initial amplitude of vibration.

Since the damping reduces the amplitude of successive free vibrations by 25%, therefore final amplitude of vibration,

$$x_2 = 0.75\, x_1$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} \quad \text{or} \quad \log_e\left(\frac{x_1}{0.75 x_1}\right) = \frac{a \times 2\pi}{\sqrt{421.6 - a^2}}$$

Squaring both sides,

$$(0.2877)^2 = \frac{a^2 \times 4\pi^2}{421.6 - a^2} \quad \text{or} \quad 0.083 = \frac{39.5\, a^2}{421.6 - a^2}$$

$$\ldots\left[\because \log_e\left(\frac{1}{0.75}\right) = \log_e 1.333 = 0.2877\right]$$

$$35 - 0.083\, a^2 = 39.5\, a^2 \quad \text{or} \quad a^2 = 0.884 \quad \text{or} \quad a = 0.94$$

We know that damping coefficient or damping force per unit velocity,

$$c = a \times 2m_1 = 0.94 \times 2 \times 100 = 188 \text{ N/m/s}$$

and critical damping coefficient,

$$c_c = 2m.\omega_n = 2 \times 100 \times 20.5 = 4100 \text{ N/m/s}$$

$\therefore$ Actual value of transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1 + \left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\dfrac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \dfrac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$= \frac{\sqrt{1 + \left(\dfrac{2 \times 188 \times 104.7}{4100 \times 20.5}\right)^2}}{\sqrt{\left(\dfrac{2 \times 188 \times 104.7}{4100 \times 20.5}\right)^2 + \left[1 - \left(\dfrac{104.7}{20.5}\right)^2\right]^2}} = \frac{\sqrt{1 + 0.22}}{\sqrt{0.22 + 629}}$$

$$= \frac{1.104}{25.08} = 0.044$$

We know that the maximum unbalanced force on the machine due to reciprocating parts,

$$F = m_2.\omega^2.r = 2(104.7)^2\,(0.08/2) = 877 \text{ N} \quad \ldots(\because \; r = l/2)$$

$\therefore$ Force transmitted to the foundation,

$$F_T = \varepsilon.F = 0.044 \times 877 = 38.6 \text{ N } \textbf{Ans.} \quad \ldots(\because \; \varepsilon = F_T / F)$$

Force transmitted to the foundation at resonance:

Since at resonance, $\omega = \omega_n$, therefore transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c}{c_c}\right)^2}}{\sqrt{\left(\frac{2c}{c_c}\right)^2}} = \frac{\sqrt{1+\left(\frac{2\times188}{4100}\right)^2}}{\sqrt{\left(\frac{2\times188}{4100}\right)^2}} = \frac{\sqrt{1+0.0084}}{0.092} = 10.92$$

and maximum unbalanced force on the machine due to reciprocating parts at resonance speed ω_n,

$$F = m_2\,(\omega_n)^2 r = 2(20.5)^2(0.08/2) = 33.6 \text{ N} \quad \ldots (\because\ r = l/2)$$

$\therefore$ Force transmitted to the foundation at resonance,

$$F_T = \varepsilon.F = 10.92\times33.6 = 367 \text{ N } \textbf{Ans.}$$

Amplitude of the forced vibration of the machine at resonance:

We know that amplitude of the forced vibration at resonance

$$= \frac{\text{Force transmitted at resonance}}{\text{Combined stiffness}} = \frac{367}{42\,160} = 8.7\times10^{-3} \text{ m}$$

$= 8.7$ mm **Ans.**

4 Single-cylinder engine of total mass 200 kg is to be mounted on an elastic support which permits vibratory movement in vertical direction only. The mass of the piston is 3.5 kg and has a vertical reciprocating motion which may be assumed simple harmonic with a stroke of 150 mm. It is desired that the maximum vibratory force transmitted through the elastic support to the foundation shall be 600 N when the engine speed is 800 r.p.m. and less than this at all higher speeds.

1. Find the necessary stiffness of the elastic support, and the amplitude of vibration at 800 r.p.m.,
2. If the engine speed is reduced below 800 r.p.m. at what speed will the transmitted force again becomes 600 N?

Given : $m1$ = 200 kg ; $m2$ = 3.5 kg ; l = 150 mm = 0.15 mm or $r = l/2$ = 0.075 m ;

Ft = 600 n ; N = 800 r.p.m. Or $\omega = 2\pi\times800 / 60 = 83.8$ rad/s

1. Stiffness of the elastic support and amplitutue vibrations:

Let s = Stiffness of elastic support in N/m, and

x_{max} = Max. amplitude of vibration in metres.

Since the max. vibratory force transmitted to the foundation is equal to the force on the elastic support (neglecting damping), therefore

Max. vibratory force transmitted to the foundation,

F_T = Force on the elastic support

= Stiffness of elastic support × Max. amplitude of vibration

$$= s \times x_{max} = s \times \frac{F}{m\left[\omega^2 - (\omega_n)^2\right]}$$

$$= s \times \frac{F}{m\left(\omega^2 - \frac{s}{m}\right)} = \frac{F.s}{m.\omega^2 - s} \qquad \ldots \left[\because (\omega_n)^2 = \frac{s}{m}\right]$$

$$600 = \frac{1843 \times s}{200\,(83.8)^2 - s} = \frac{1843\,s}{1.4 \times 10^6 - s} \qquad \ldots \text{(Substituting } m = m_1\text{)}$$

We know that the disturbing force at 800 r.p.m., F = centrifugal force on the piston.

or $\quad 840 \times 10^6 - 600\,s = 1843\,s$

$\therefore \quad s = 0.344 \times 10^6 = 344 \times 10^3$ N/m **Ans.**

and maximum amplitude of vibration,

$$x_{max} = \frac{F}{m.\omega^2 - s} = \frac{1843}{200\,(83.8)^2 - 344 \times 10^3} = \frac{1843}{1056 \times 10^3}\ \text{m}$$

$$= 1.745 \times 10^{-3}\ \text{m} = 1.745\ \text{mm}\ \textbf{Ans.}$$

Speed at the which the transmitted force again becomes 600N

The transmitted force will rise as the speed of the engine falls and passes through resonance. There will be a speed below resonance at which the transmitted force will again equal to 600 N. Let this speed be ω_1 rad/s (or N_1 r.p.m.).

$\therefore$ Disturbing force, $F = m_2\,(\omega_1)^2\,r = 3.5\,(\omega_1)^2\,0.075 = 0.2625\,(\omega_1)^2$ N

Since the engine speed is reduced below N_1 = 800 r.p.m., therefore in this case, max. amplitude of vibration,

$$x_{max} = \frac{F}{m\left[(\omega_n)^2 - (\omega_1)^2\right]} = \frac{F}{m\left[\frac{s}{m} - (\omega_1)^2\right]} = \frac{F}{s - m\,(\omega_1)^2}$$

and $\quad$ Force transmitted $= s \times \dfrac{F}{s - m\,(\omega_1)^2}$

$$\therefore \quad 600 = 344 \times 10^3 \times \frac{0.2625\,(\omega_1)^2}{344 \times 10^3 - 200\,(\omega_1)^2} = \frac{90.3 \times 10^3\,(\omega_1)^2}{344 \times 10^2 - 200\,(\omega_1)^2}$$

$\ldots$ (Substituting $m = m_1$)

$206.4 \times 10^6 - 120 \times 10^3\,(\omega_1)^2 = 90.3 \times 10^3\,(\omega_1)^2 \quad$ or $\quad (\omega_1)^2 = 981$

$\therefore \quad \omega_1 = 31.32$ rad/s or $N_1 = 31.32 \times 60 / 2\pi = 299$ r.p.m. **Ans.**

5. A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one-tenth of the initial value in four complete oscillations. If a periodic force of **150 cos 50 t** N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations. What is its value of resonance?

Given:

m = 10 kg; s = 10 N/mm = 10 × 103N/m ; X5 = X1 / 10

Since the periodic force, $F_x = F\cos\omega t = 150\cos 50t$, therefore

Static force, $F = 150$ N

and angular velocity of the periodic disturbing force,

$$\omega = 50 \text{ rad/s}$$

We know that angular speed or natural circular frequency of free vibrations,

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_5}\right)^{1/4} = \left(\frac{x_1}{x_1/10}\right)^{1/4} = (10)^{1/4} = 1.78$$

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e 1.78 = a \times \frac{2\pi}{\sqrt{(31.6)^2 - a^2}} \quad \text{or} \quad 0.576 = \frac{a \times 2\pi}{\sqrt{1000 - a^2}}$$

Squaring both sides and rearranging,

$39.832\, a^2 = 332$ or $a^2 = 8.335$ or $a = 2.887$

We know that $a = c/2m$ or $c = a \times 2m = 2.887 \times 2 \times 10 = 57.74$ N/m/s

and deflection of the system produced by the static force F,

$$x_o = F/s = 150/10 \times 10^3 = 0.015 \text{ m}$$

We know that amplitude of the forced vibrations,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left[1 - \frac{\omega^2}{(\omega_n)^2}\right]^2}}$$

$$= \frac{0.015}{\sqrt{\frac{(57.74)^2(50)^2}{(10\times10^3)^2} + \left[1 - \left(\frac{50}{31.6}\right)^2\right]^2}} = \frac{0.015}{\sqrt{0.083 + 2.25}}$$

$$= \frac{0.015}{1.53} = 9.8\times10^{-3} \text{ m} = 9.8 \text{ mm } \textbf{Ans.}$$

Amplitute of forced vibrations:

We know that amplitude of forced vibrations at resonance,

$$x_{max} = x_0 \times \frac{s}{c.\omega_n} = 0.015 \times \frac{10 \times 10^3}{57.54 \times 31.6} = 0.0822 \text{ m} = 82.2 \text{ mm} \quad \textbf{Ans.}$$

6. The mass of an electric motor is 120 kg and it runs at 1500 r.p.m. The armature mass is 35 kg and its C.G. lies 0.5 mm from the axis of rotation. The motor is mounted on five springs of negligible damping so that the force transmitted isone-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs. Determine: stiffness of each spring; **2.** dynamic force transmitted to the base at the operating speed; and **3.** natural frequency of the system

.Given *m*1 = 120 kg; *m*2 = 35 kg; *r* = 0.5 mm = 5 × 10–4 m; ϵ = 1 / 11; *N* = 1500 r.p.m. or

1. ***Stiffness of each spring***

Let *s* = Combined stiffness of the spring in N-m, and □*n* = Natural circular frequency of vibration of the machine in rad/s. We know that transmissibility ratio (ϵ)

$$\frac{1}{11} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(157.1)^2 - (\omega_n)^2}$$

$$(157.1)^2 - (\omega_n)^2 = 11(\omega_n)^2 \quad \text{or} \quad (\omega_n)^2 = 2057 \quad \text{or} \quad \omega_n = 45.35 \text{ rad/s}$$

We know that $\omega_n = \sqrt{s / m_1}$

$$s = m_1 (\omega_n)^2 = 120 \times 2057 = 246\,840 \text{ N/m}$$

Since these are five springs, therefore stiffness of each spring

$$= 246\,840 / 5 = 49\,368 \text{ N/m} \quad \textbf{Ans.}$$

1. Dynamics force transmitted to the base at the operating speed:

We know that maximum unbalanced force on the motor due to armature mass,

$$F = m_2\, \omega^2 \cdot r = 35(157.1)^2\, 5 \times 10^{-4} = 432 \text{ N}$$

∴ Dynamic force transmitted to the base,

$$F_T = \varepsilon . F = \frac{1}{11} \times 432 = 39.27 \text{ N} \quad \textbf{Ans.}$$

2. Natural frequency of the system:

We have calculated above that the natural frequency of the system,

$$\omega_n = 45.35 \text{ rad/s} \quad \textbf{Ans.}$$

UNIT-V MECHNISMS FOR CONTROL

Types of Mechanisms for control are

1) Governors.
2) Gyroscope.

Governors:

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

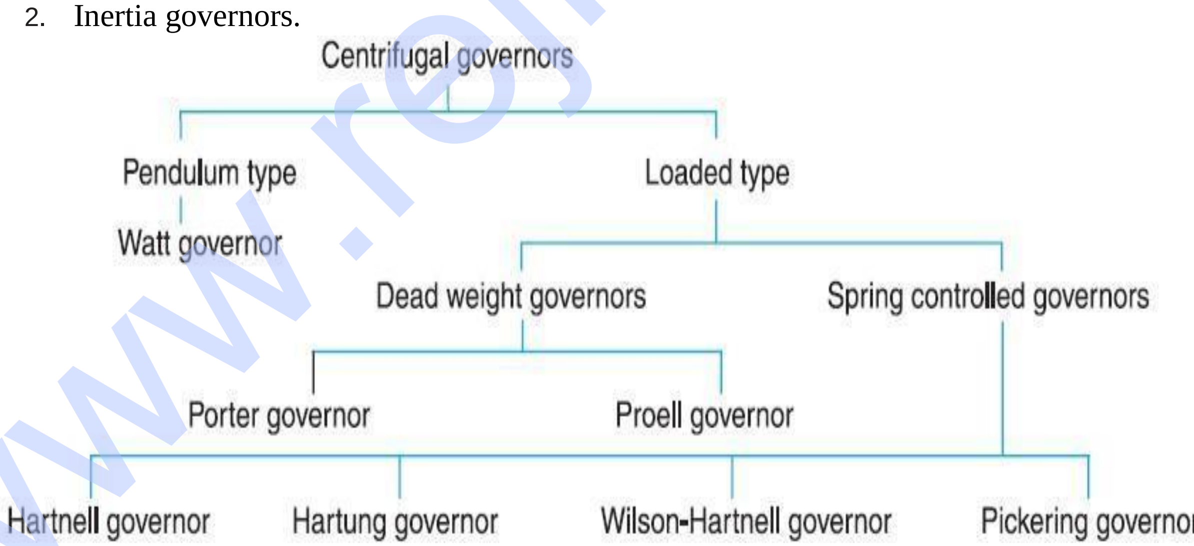
Difference between a Flywheel and Governor:

The function of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

Types of Governors

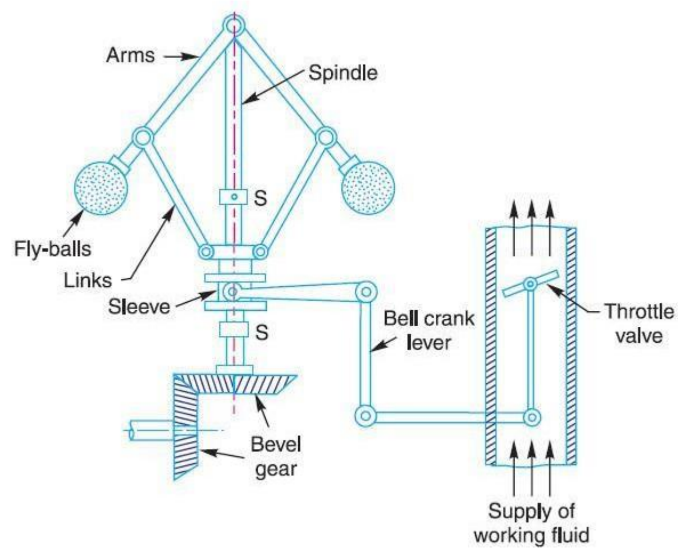
The governors may, broadly, be classified as

1. Centrifugal governors
2. Inertia governors.



Centrifugal governors:

The centrifugal governors are based on the balancing of centrifugal force on the rotating ballsby an equal and opposite radial force, known as the controlling force.It consists of two balls ofequal mass, which are attached to the arms as shown in Fig. These balls are known as governorballs or fly balls.



The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about

thevertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops S, S are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Terms Used in Governors

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.
2. Equilibrium speed. It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.
3. Mean equilibrium speed. It is the speed at the mean position of the balls or the sleeve.
4. Maximum and minimum equilibrium speeds. The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. Sleeve lift. It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Porter Governor:

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig.(a). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level. Consider the forces acting on one-half of the governor as shown in Fig.(b).

m = Mass of each ball in kg,
w = Weight of each ball in newtons = m.g,
M = Mass of the central load in kg,
W = Weight of the central load in newtons = M.g,
r = Radius of rotation in metres,
h = Height of governor in metres,

N = Speed of the balls in r.p.m.
 = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,
FC = Centrifugal force acting on the ball in newtons = $m.$ 2.r,
$T1$ = Force in the arm in newtons,
$T2$ = Force in the link in newtons,
α = Angle of inclination of the arm (or upper link) to the vertical, and = Angle of inclination of the link (or lower link) to the vertical.

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

1. When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then
   tanα = tanβ or q = tanα / tanβ = 1

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

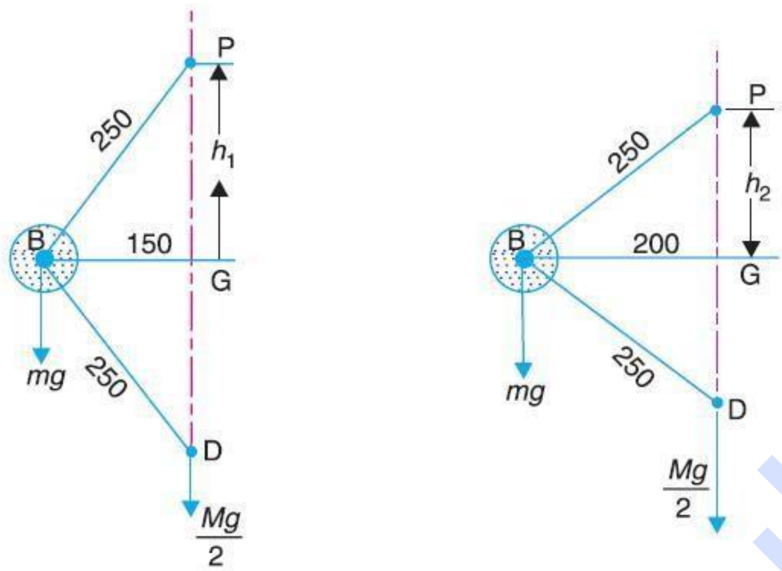
2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.
If F = Frictional force acting on the sleeve in newtons, then the equations (v) and (vi) may be written as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

1. A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. Theradius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.

Gn :$BP = BD$ = 250 mm = 0.25 m ; m = 5 kg ; M = 15 kg ; r_1 = 150 mm = 0.15m; r_2 = 200 mm = 0.2 m



(*a*) Minimum position. (*b*) Maximum position.

Minimum speed when r1 = BG = 0.15 m

Let $N1$ = Minimum speed.

From Fig(*a*), we find that height of the governor,

$h = PG = \sqrt{((PB)\wedge 2 - (BG)\wedge 2)} = 0.2$ m

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+15}{5} \times \frac{895}{0.2} = 17\,900$$

N_1 = 133.8 r.p.m.

Maximum speed when r_2 = BG = 0.2 m

Let N_2 = Maximum speed.

From Fig(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+15}{5} \times \frac{895}{0.15} = 23\,867$$

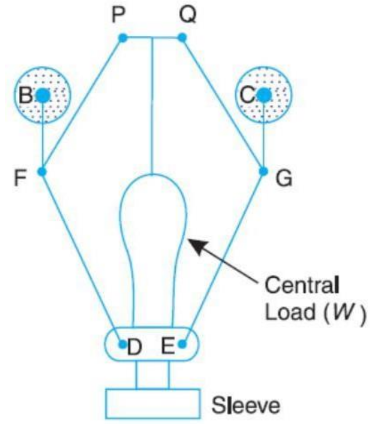
N_2 = 154.5 r.p.m

Range of speed

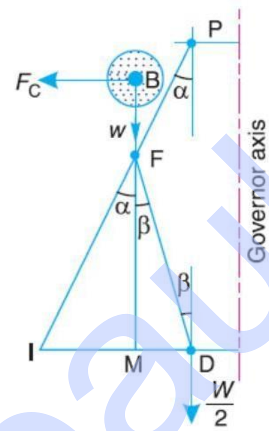
We know that range of speed= $N_2 - N_1$ = 154.4 – 133.8 = **20.7 r.p.m.**

Proell governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively. Consider the equilibrium of the forces on one-half of the governor as shown in Fig (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *BM* is drawn on *ID*.



(*a*)



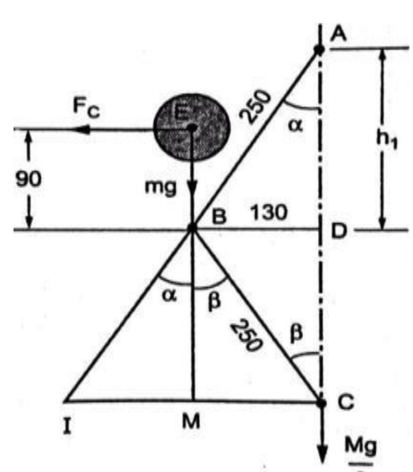
(*b*)

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

1. A proell governor has equal arms of length 250mm.the upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 90mm long and parallel to the axis. When the radii of rotation of the balls are 130mm and 175mm. The mass of each ball is 8.5 kg and the mass of the central load is 85 kg. Determine the range of speed of the governor

Given data:

AB=BC=250mm,
EB=90mm,
r_1=130mm,
r_2=175mm,
m=8.5kg,
M=85kg



For minimum speed when r_1=130mm h_1 = AD =$\sqrt{AB^2 - BD^2}$

$\sqrt{0.25^2 - 0.13^2}$

=0.2135m

BM=DC=AD=0.2135m

α=β so q = 1

$N^2_1 \quad \frac{BM}{EM}[\frac{m+M}{m}] \quad (895/h_1)$

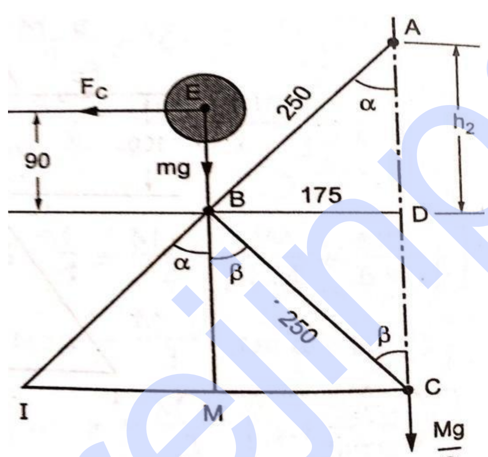
$\frac{0.2135}{0.3035}[\frac{8.5+85}{8.5}] \quad (895/0.2135)$

=32438.2

N_1 =180.1 rpm

For maximum speed when r_2=175mm

h_2 = AD =$\sqrt{AB^2 - BD^2}$



$\sqrt{0.25^2 - 0.175^2}$

=0.1785m

BM=DC=AD=0.1785m

EM=EB+BM

=0.09+0.1785

=0.2685m

α=β so q = 1

$N^2_2 \quad \frac{BM}{EM}[\frac{m+M}{m}] \quad (895/h_2)$

$\frac{0.1785}{0.2658}[\frac{8.5+85}{8.5}]$

=36666.67

N_1 =191.48 rpm

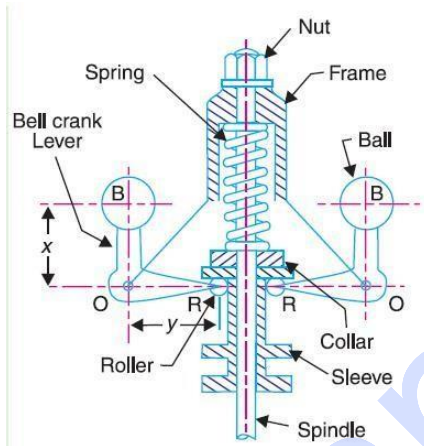
Range of speed = maximum speed – minimum speed

=191.48-180.1

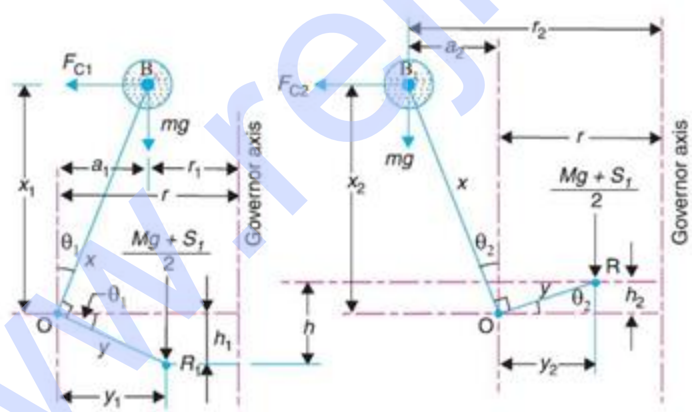
=11.38rpm

Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 18.18. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



Hartnell governor



(a) Minimum position. (b) Maximum position.

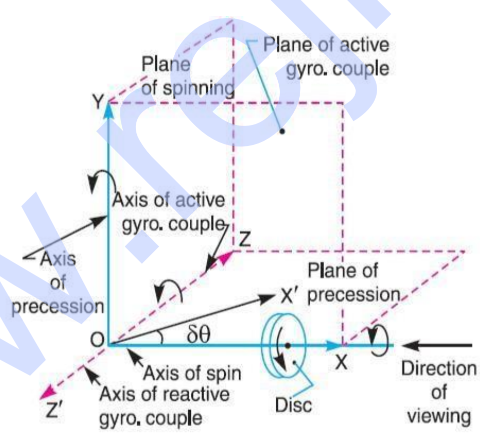
Gyroscopic Couple:

Consider a disc spinning with an angular velocity ω rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig(a). Since the plane in which the disc is rotating is parallel to the plane YOZ, therefore it is called plane of spinning. The planeXOZ is a horizontal plane and the axis of spin rotates in a plane parallel to the horizontal plane aboutan axis OY. In other words, the axis of spin is said to be rotating or processing about an axis OY. Inother words, the axis of spin is said to be rotating or processing about an axis OY (which is perpendicular to both the axes OX and OZ) at an angular velocity ω_P rap/s. This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

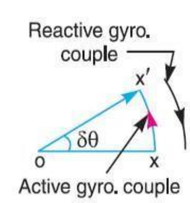
I = Mass moment of inertia of the disc about *OX,* andω = Angular velocity of the disc.

Angular momentum of the disc= $I.\omega$

Since the angular momentum is a vector quantity, therefore it may be represented by the vector $\underline{ox}$, as shown in Fig.(*b*). The axis of spin *OX* is also rotating anticlockwise when seen from the top about the axis *OY*. Let the axis *OX* is turned in the plane *XOZ* through a small angle δθ radians to the position*OX'* , in time δ*t*seconds. Assuming the angular velocity ω to be constant, the angular momentum will now be represented by vector *OX'*.



(*a*)



(*b*)

Change in angular momentum = I.ω.δθ

Rate of change of angular momentum

$$= I.\omega \times \frac{\delta\theta}{dt}$$

The rate of change of angular momentum will result by the application of a couple to the disc, therefore the couple applied to the disc causing precession

$$C = I.\omega.\omega_P$$

1. A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis?

Given: d = 300 mm or r = 150 mm = 0.15 m ; m = 5 kg ; l = 600 mm = 0.6 m ; N = 300 r.p.m. or $\omega = 2\pi \times 300/60$ = 31.42 rad/s.

$I = m.r^2/2 = 5(0.15)^2/2 = 0.056$ kg-m^2

$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43$ N-m

ω_P = Speed of precession.

We know that couple (C),

$$29.43 = I.\omega.\omega_P = 0.056 \times 31.42 \times \omega_P = 1.76\ \omega_P$$

$$\therefore \quad \omega_P = 29.43/1.76 = 16.7 \text{ rad/s} \quad \textbf{Ans.}$$

Effect of the Gyroscopic Couple on an Aeroplane

The top and front views of an aeroplane are shown in Fig (a). Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.

Notes: 1. when the aeroplane takes a right turn under similar conditions as discussed above, the effect of the reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.

2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.

3. When the aeroplane takes a right turn under similar conditions as mentioned in note 2 above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.

4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the nose of the aeroplane.

5. When the aeroplane takes a right turn under similar conditions as mentioned in note 4-above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.

2. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.

Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ;

N = 2400 r.p.m. or □ = 2π × 2400/60 = 251 rad/s

We know that mass moment of inertia of the engine and the propeller,

$I = m.k2$= 400(0.3)2= 36 kg-m2

and angular velocity of precession,

□P= v/R= 55.6/50 = 1.11 rad/s

We know that gyroscopic couple acting on the aircraft,

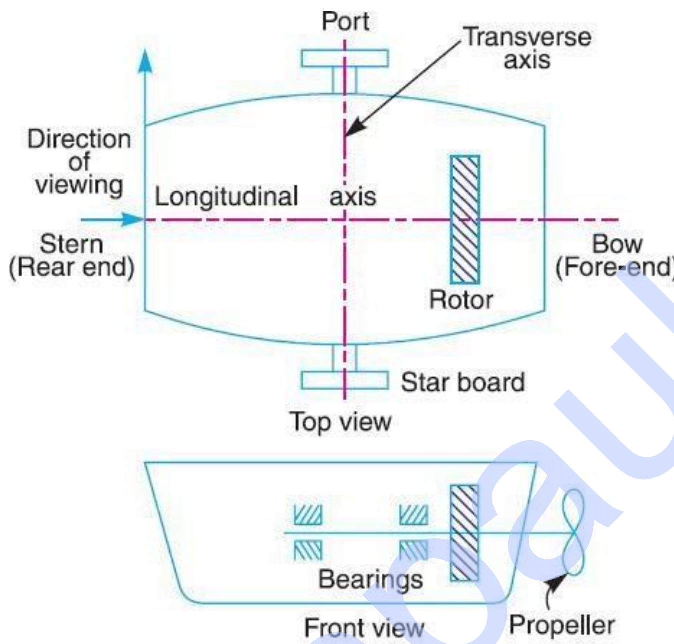
$C = I.$□□□P= 36 × 251.4 × 1.11 = 100 46 N-m = **10.046 kN-m**

When the aeroplane turns towards left, the effect of couple is to lift the nose upwards and tail downwards.

Terms Used in a Naval Ship

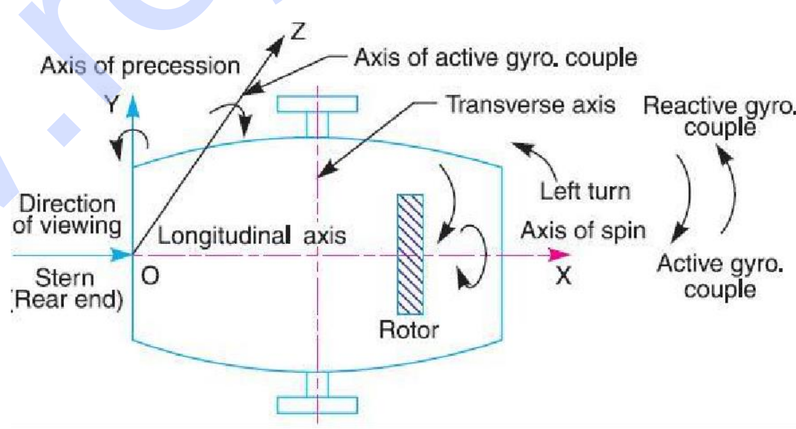
The top and front views of a naval ship are shown in Fig. The fore end of the ship iscalled bow and the rear end is known as stern or aft. The left hand and right hand sides of the ship,when viewed from the stern are called port and star-board respectively. We shall now discuss theeffect of gyroscopic couple on the naval ship in the following three cases:

1. Steering, 2. Pitching and 3. Rolling.

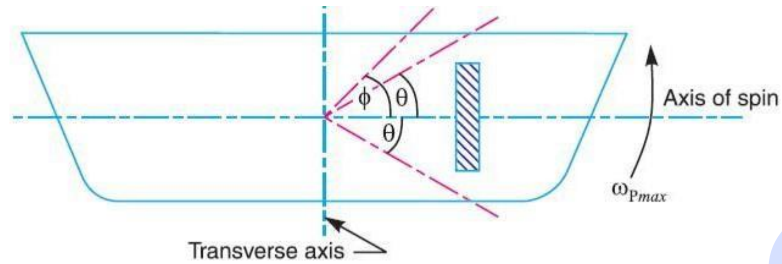


Effect of Gyroscopic Couple on a Naval Ship during Steering

Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane as discussed earlier.



<u>Effect of Gyroscopic Couple on a Naval Ship during Pitching</u>



<u>Effect of Gyroscopic Couple on a Naval Ship during Rolling</u>

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship. In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

1. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

1. When the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.

2. When the ship is pitching in a simple harmonic motion, the bow falling with its maximum velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Given: m = 3500 kg; k = 0.45 m; N = 3000 r.p.m. or □ □ □π × 3000/60 = 314.2 rad/s

1. When the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.

Given: R = 100 m ; v = km/h = 10 m/s

We know that mass moment of inertia of the rotor,

$$I = m.k^2 = 3500\,(0.45)^2 = 708.75 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 10/100 = 0.1 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_p = 708.75 \times 314.2 \times 0.1 = 22\,270 \text{ N-m}$$

$$= 22.27 \text{ kN-m}$$ **Ans.**

2. When the ship is pitching

Given: $t_p = 40$ s

Since the total angular displacement between the two extreme positions of pitching is 12° (*i.e.* $2\phi = 12°$), therefore amplitude of swing,

$$\phi = 12/2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

2.The turbine rotor of a ship has a 2.4 tonnes and rotates at 1750 rpm when viewed from the left. The radius of gyration of the rotor is 300mm.Determine gyroscopic couple and its effect when

(i) The ship turns right at a radius of 250m with a speed of 22kmph.

(ii) The ship pitches with the bow rising at an angular velocity of 0.85 rad/sec.

(iii) The ship rolls at an angular velocity of 0.15rad/sec

Given data:

m = 2.4 t = 2400kg, N=1750rpm , k= 300mm , v= 22kmph = 22*(5/18)=6.11m/s

R = 250m.

Solution:

Mass moment of inertia $I = mk^2$

$= 2400(0.3)^2$

$= 216 \text{ kg-m}^2$

Angular velocity(ω)

$$\omega = 2\pi N/60$$

$$\omega = 2\pi(1750)/60$$

$= 183.26$ rad/sec

1. When ship takes right turn

Angular velocity of precession

(ω_p) $\omega_p = V / R = 6.11 / 250$

$= 0.0244$ rad

/sec Gyroscopic couple

(C)

$C = I\,\omega\,\omega_p$

$C = I\,\omega\,\omega_p$

$=216(183.26)(0.15)$

$=5937.6$N-m

As the axis of spin is always parallel to the axis of precession for all position, there is no gyroscopic effect on the ship.

3.A ship propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 r.p.m. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effects in the following conditions:

a. The ship sails at a speed of 30 km/h and steers to the left in a curve having 60 m radius.

b. The ship pitches 6 degree above and 6 degree below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds.

c. The ship rolls and at a certain instant it has an angular velocity of 0.03 rad/s clockwise when viewed from stern. Determine also the maximum angular acceleration during pitching. Explain how the direction of motion due to gyroscopic effect is determined in each case.

Solution. Given : m = 5 t = 5000 kg ; N = 2100 r.p.m. or $\omega = 2\pi \times 2100/60 = 220$ rad/s ; k = 0.5 m

1. *When the ship steers to the left*

Given: v = 30 km / h = 8.33 m / s ; R = 60 m

We know that angular velocity of precession,

$$\omega_P = v/R = 8.33/60 = 0.14 \text{ rad/s}$$

and mass moment of inertia of the rotor,

$$I = m.k^2 = 5000(0.5)^2 = 1250 \text{ kg-m}^2$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.14 = 38\,500 \text{ N-m} = 38.5 \text{ kN-m}$$

We have discussed in Art. 14.6, that when the rotor in a clockwise direction when viewed from the stern and the ship steers to the left, the effect of reactive gyroscopic couple is to raise the bow and lower the stern. **Ans.**

2. *When the ship pitches with the bow descending*

Given: $\phi = 6° = 6 \times \pi/180 = 0.105$ rad/s ; t_p = 20 s

We know that angular velocity of simple harmonic motion,

$$\omega_1 = 2\pi / t_p = 2\pi / 20 = 0.3142 \text{ rad/s}$$

and maximum angular velocity of precession,

$$\omega_{Pmax} = \phi.\omega_1 = 0.105 \times 0.3142 = 0.033 \text{ rad/s}$$

∴ Maximum gyroscopic couple,

$$C_{max} = I.\omega.\omega_{Pmax} = 1250 \times 220 \times 0.033 = 9075 \text{ N-m}$$

Since the ship is pitching with the bow descending, therefore the effect of this maximum gyroscopic couple is to turn the ship towards port side. **Ans.**

3. *When the ship rolls*

Since the ship rolls at an angular velocity of 0.03 rad / s, therefore angular velocity of precession when the ship rolls,

$$\omega_P = 0.03 \text{ rad /s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\omega_P = 1250 \times 220 \times 0.03 = 8250 \text{ N-m}$$

In case of rolling of a ship, the axis of precession is always parallel to the axis of spin for all positions, therefore there is no effect of gyroscopic couple. **Ans.**

Maximum angular acceleration during pitching

We know that maximum angular acceleration during pitching.

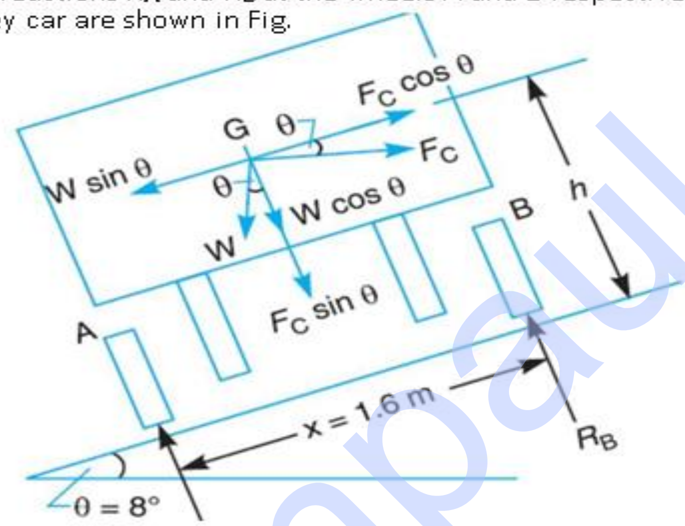
$$\alpha_{max} = \phi\,(\omega_1)^2 = 0.105\,(0.3142)^2 = 0.01 \text{ rad/s}^2 \textbf{ Ans.}$$

Stability of a Four Wheel Drive Moving in a Curved Path

4. A four-wheeled trolley car of total mass 2000 kg running on rails of 1.6 m gauge, rounds a curve of 30 m radius at 54 km/h. The track is banked at 8°. The wheels have an external diameter of 0.7 m and each pair with axle has a mass of 200 kg. The radius of gyration for each pair is 0.3 m. The height of centre of gravity of the car above the wheel base is 1 m. Determine, allowing for centrifugal force and gyroscopic couple actions, the pressure on each rail.

Given: m = 2000 kg ; x = 1.6 m ; R = 30 m ; v = 54 km / h = 15 m / s ; θ = 8° ; d_W = 0.7 m or r_W = 0.35 m ; m_1 = 200 kg ; k = 0.3 m ; h = 1 m

First of all, let us find the reactions R_A and R_B at the wheels A and B respectively. The various forces acting on the trolley car are shown in Fig.



$$= (19\,430 + 2088)\,\frac{1}{2} + (2731 - 14\,855)\,\frac{1}{1.6}$$

$$= 10\,759 - 7577 = 3182 \text{ N}$$

$$\therefore \quad R_B = (R_A + R_B) - R_A = 21\,518 - 3182 = 18\,336 \text{ N}$$

We know that angular velocity of wheels,

$$\omega_W = \frac{v}{r_W} = \frac{15}{0.35} = 42.86 \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{15}{30} = 0.5 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I\,\omega_W \cos\theta \times \omega_P = m_1 k^2.\omega_W \cos\theta.\omega_P \qquad \text{...}(\because I = m_1 k^2)$$

$$= 200\,(0.3)^2\,42.86 \cos 8° \times 0.5 = 382 \text{ N-m}$$

Due to this gyroscopic couple, the car will tend to overturn about the outer wheels. Let P be the force at each pair of wheels or each rail due to the gyroscopic couple,

$$\therefore \quad P = C / x = 382 / 1.6 = 238.75 \text{ N}$$

We know that pressure (or total reaction) on the inner rail,

$$P_I = R_A - P = 3182 - 238.75 = 2943.25 \text{ N} \quad \textbf{Ans.}$$

and pressure on the outer rail,

$$P_O = R_B + P = 18\,336 + 238.75 = 18\,574.75 \text{ N} \quad \textbf{Ans.}$$

5.The driving axle of a locomotive with two wheels has a moment of inertia of 175 kg/m^2.The diameter of the wheel treads is 1.7m and the distance between wheel centers is 1.5m. When the locomotive is travelling on a level track @ 88km/hr, defective ballasting causes one wheel to fall 5mm and rise again in a total time of 1sec, if the displacement of the wheel takes place with simple harmonic motion. Find

1. Gyroscopic couple setup

2. The reaction between the wheel and rail due to this couple.

Given data:

$I = 175kg/m^2$,$d_w = 1.7m$, x=1.5m, v = 88km/hr =88*(5/18) =24.44m/s t_p=0.1 sec

fall = 0.005mm

Solution:

1. Gyroscopic couple setup(C)

Angular velocity (ω)

$= V / r_w$

=24.44 / 0.85

=28.75 rad/sec

Since the defective ballasting causes one wheel to fall 5mm and rise again in a total time of 0.1sec

Therefore amplitude A = ½(fall)= ½(rise)

= ½(0.005)

=0.0025 mm

Maximum velocity while falling

$V_{max}=(2\pi/tp)A$

$=(2\pi/0.1)0.0025$

=0.15

7 m/s Gyroscopic couple setup (C)

$C = I\ \omega\ \omega_p$

=175(28.75)(0.105)

=528.28 N-m

2. <u>Reaction between the wheel and rail due to the gyroscopic couple.</u>

P = C/x = 528.18 / 1.5

= 352.2 N

6. A four-wheeled trolley car of mass 2500 kg runs on rails, which are 1.5 m apart and travels around a curve of 30 m radius at 24 km / hr. The rails are at the same level. Each wheel of the trolley is 0.75 m in diameter and each of the two axles is driven by a motor running in a direction opposite to that of the wheels at a speed of five times the speed of rotation of the wheels. The moment of inertia of each axle with gear and wheels is 18 kg-m^2. Each motor with shaft and gear pinion has a moment of inertia of 12 kg-m^2. The centre of gravity of the car is 0.9 m above the rail level. Determine the vertical force exerted by each wheel on the rails taking into consideration the centrifugal and gyroscopic effects. State the centrifugal and gyroscopic effects on the trolley.

Solution. Given : m= 2500 kg ; x= 1.5 m ; R= 30 m ; v= 24 km/h = 6.67 m/s ; d_W= 0.75 m or r_W= 0.375 m ; G= ω_E/ω_W= 5 ; I_W= 18 kg-m^2;I_E= 12 kg-m^2 ; h= 0.9 m

The weight of the trolley (W= m.g) will be equally distributed over the four wheels, which will act downwards. The reaction between the wheels and the road surface of the same magnitude will act upwards.

$\therefore$ Road reaction over each wheel $= W/4 = m.g/4 = 2500 \times 9.81/4 = 6131.25$ N

We know that angular velocity of the wheels,

$$\omega_W = v/r_W = 6.67/0.375 = 17.8 \text{ rad/s}$$

and angular velocity of precession, $\omega_P = v/R = 6.67/30 = 0.22$ rad/s

$\therefore$ Gyroscopic couple due to one pair of wheels and axle,

$$C_W = 2\, I_W.\omega_W.\omega_P = 2 \times 18 \times 17.8 \times 0.22 = 141 \text{ N-m}$$

and gyroscopic couple due to the rotating parts of the motor and gears,

$$C_E = 2\, I_E.\omega_E.\omega_P = 2\, I_E.G.\omega_W.\omega_P \quad \ldots (\because\ \omega_E = G.\,\omega_W)$$

$$= 2 \times 12 \times 5 \times 17.8 \times 0.22 = 470 \text{ N-m}$$

$\therefore$ Net gyroscopic couple, $C = C_W - C_E = 141 - 470 = -329$ N-m

... (–ve sign is used due to opposite direction of motor)

Due to this net gyroscopic couple, the vertical reaction on the rails will be produced. Since C_E is greater than C_W , therefore the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheel be P/2 newton.

$\therefore$ $P/2 = C/2x = 329 / 2 \times 1.5 = 109.7$ N

We know that centrifugal force, $F_C = m.v^2/R = 2500\,(6.67)^2/30 = 3707$ N

$\therefore$ Overturning couple, $C_O = F_C \times h = 3707 \times 0.9 = 3336.3$ N-m

This overturning couple is balanced by the vertical reactions which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be Q/2 newton.

$$Q/2 = C_O / 2x = 3336.3 / 2 \times 1.5 = 1112.1 \text{ N}$$

We know that vertical force exerted on each outer wheel,

$$P_O = \frac{W}{4} - \frac{P}{2} + \frac{Q}{2} = 6131.25 - 109.7 + 1112.1 = 7142.65 \text{ N}$$ **Ans.**

and vertical force exerted on each inner wheel,

$$P_I = \frac{W}{4} + \frac{P}{2} - \frac{Q}{2} = 6131.25 + 109.7 - 1112.1 = 5128.85 \text{ N}$$ **Ans.**

7.A rear engine automobile is travelling along a track of 100 metres mean radius. Each of the four road wheels has a moment of inertia of 2.5 kg-m^2 and an effective diameter of 0.6 m. The rotating parts of the engine have a moment of inertia of 1.2 kg-m^2. The engine axis is parallel to the rear axle and the crankshaft rotates in the same sense as the road wheels. The ratio of engine speed to back axle speed is 3 : 1. The automobile has a mass of 1600 kg and has its centre of gravity 0.5 m above road level. The width of the track of the vehicle is 1.5 m. Determine the limiting speed of the vehicle around the curve for all four wheels to maintain contact with the road surface. Assume that the road surface is not cambered and centre of gravity of the automobile lies centrally with respect to the four wheels.

Given : R= 100 m ; I_W= 2.5 kg-m^2; d_W= 0.6 m or r_W= 0.3 m ; I_E= 1.2 kg-m^2; G= ω_E/ω_W= 3 ; m= 1600 kg ; h= 0.5 m ; x= 1.5 m

The weight of the vehicle (m.g) will be equally distributed over the four wheels which will act downwards. The reaction between the wheel and the road surface of the same magnitude will act upwards.

∴Road reaction over each wheel

$$= W/4 = m.g/4 = 1600 \times 9.81/4 = 3924 \text{ N}$$

Let v = Limiting speed of the vehicle in m/s.

We know that angular velocity of the wheels,

$$\omega_W = \frac{v}{r_W} = \frac{v}{0.3} = 3.33\, v \text{ rad/s}$$

and angular velocity of precession,

$$\omega_P = \frac{v}{R} = \frac{v}{100} = 0.01\, v \text{ rad/s}$$

∴ Gyroscopic couple due to 4 wheels,

and gyroscopic couple due to rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P$$
$$= 1.2 \times 3 \times 3.33v \times 0.01v = 0.12\, v^2 \text{ N-m}$$

∴ Total gyroscopic couple,

$$C = C_W + C_E = 0.33\, v^2 + 0.12\, v^2 = 0.45\, v^2 \text{ N-m}$$

Due to this gyroscopic couple, the vertical reaction on the rails will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheel be P/2 newtons.

$$\therefore \quad P/2 = C/2x = 0.45v^2/2 \times 1.5 = 0.15\, v^2 \text{ N}$$

We know that centrifugal force,

$$F_C = m.v^2/R = 1600 \times v^2/100 = 16\, v^2 \text{ N}$$

∴Overturning couple acting in the outward direction,

$$C_O = F_C \times h = 16\, v^2 \times 0.5 = 8\, v^2 \text{ N-m}$$

This overturning couple is balanced by vertical reactions which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be Q/2 newtons.

$\therefore \qquad Q/2 = C_O/2x = 8\,v^2/2 \times 1.5 = 2.67\,v^2$ N

We know that total vertical reaction at each of the outer wheels,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheels,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2} = \frac{W}{4} - \left(\frac{P}{2} + \frac{Q}{2}\right)$$

From equation (i),we see that there will always be contact between the outer wheels and the road surface because W/4, P/2 and Q/2 are vertically upwards. In order to have contact between the inner wheels and road surface, the reactions should also be vertically upwards, which is only possible if

$$\frac{P}{2} + \frac{Q}{2} \leq \frac{W}{4}$$

i.e. $\qquad 0.15\,v^2 + 2.67\,v^2 \leq 3924 \qquad$ or $\qquad 2.82\,v^2 \leq 3924$

$\therefore \qquad v^2 \leq 3924/2.82 = 1391.5$

or $\qquad v \leq 37.3$ m/s $= 37.3 \times 3600 / 1000 = 134.28$ km/h **Ans.**